CHAPTER 2
Polynomial and Rational Functions

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Section 2.1 Quadratic Functions

You should know the following facts about parabolas.

- $f(x) = ax^2 + bx + c$, $a \neq 0$, is a quadratic function, and its graph is a parabola.
- If $a > 0$, the parabola opens upward and the vertex is the minimum point. If $a < 0$, the parabola opens downward and the vertex is the maximum point.
- The vertex is $(-b/2a, f(-b/2a))$.
- To find the x-intercepts (if any), solve
  $ax^2 + bx + c = 0.$
- The standard form of the equation of a parabola is
  $f(x) = a(x - h)^2 + k$
  where $a \neq 0$.
  (a) The vertex is (h, k).
  (b) The axis is the vertical line $x = h$.

Vocabulary Check

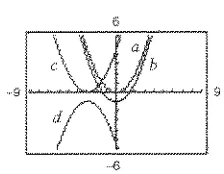
1. nonnegative integer, real
2. quadratic, parabola
3. axis
4. positive, minimum
5. negative, maximum

1. $f(x) = (x - 2)^2$ opens upward and has vertex $(2, 0)$. Matches graph (c).

2. $f(x) = 3 - x^2$ opens downward and has vertex $(0, 3)$. Matches graph (d).

3. $f(x) = x^2 + 3$ opens upward and has vertex $(0, 3)$. Matches graph (b).

4. $f(x) = -(x - 4)^2$ opens downward and has vertex $(4, 0)$. Matches graph (a).

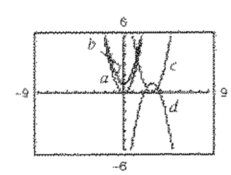
5. 

(a) $y = \frac{1}{2}x^2$, vertical shrink

(b) $y = \frac{1}{2}x^2 - 1$, vertical shrink and vertical shift one unit downward

(c) $y = \frac{1}{2}(x + 3)^2$, vertical shrink and horizontal shift three units to the left

(d) $y = -\frac{1}{2}(x + 3)^2 - 1$, horizontal shift three units to the left, vertical shrink, reflection in x-axis, and vertical shift one unit downward

6. 

(a) $y = \frac{3}{2}x^2$, vertical stretch

(b) $y = \frac{3}{2}x^2 + 1$, vertical stretch, followed by a vertical shift upward one unit

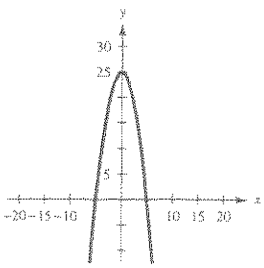
(c) $y = \frac{3}{2}(x - 3)^2$, horizontal shift three units to the right, followed by a vertical stretch

(d) $y = -\frac{3}{2}(x - 3)^2 + 1$, horizontal shift three units to the right, a vertical stretch, a reflection in the x-axis, and a vertical shift one unit upward

7. $f(x) = 25 - x^2$

Vertex: $(0, 25)$

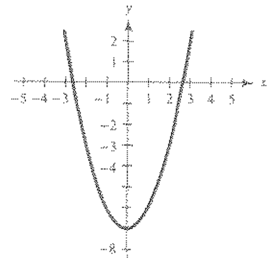
x-intercepts: $(-5, 0), (5, 0)$



8. $f(x) = x^2 - 7$

Vertex: $(0, -7)$

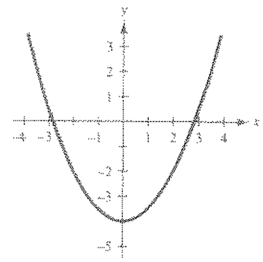
Intercepts: $(\pm\sqrt{7}, 0)$



9. $f(x) = \frac{1}{2}x^2 - 4$

Vertex: $(0, -4)$

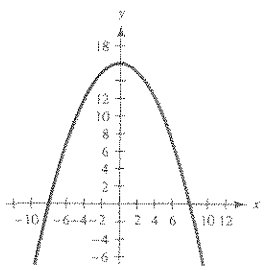
x-intercepts: $(\pm 2\sqrt{2}, 0)$



10. $f(x) = 16 - \frac{1}{4}x^2$

Vertex: $(0, 16)$

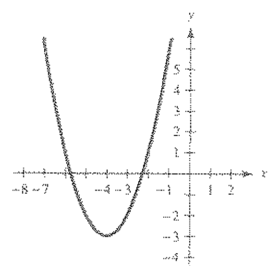
Intercepts: $(\pm 8, 0)$



11. $f(x) = (x + 4)^2 - 3$

Vertex: $(-4, -3)$

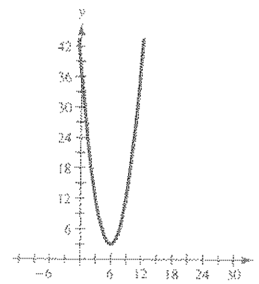
x-intercepts: $(-4 \pm \sqrt{3}, 0)$



12. $f(x) = (x - 6)^2 + 3$

Vertex: $(6, 3)$

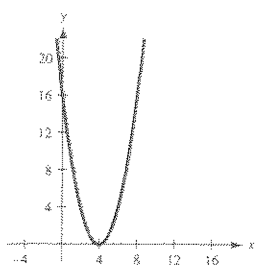
No x-intercepts



13. $h(x) = x^2 - 8x + 16 = (x - 4)^2$

Vertex: $(4, 0)$

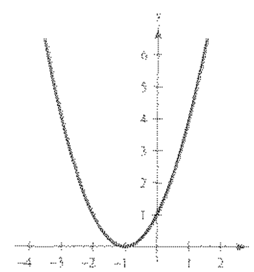
x-intercepts: $(4, 0)$



14. $g(x) = x^2 + 2x + 1 = (x + 1)^2$

Vertex: $(-1, 0)$

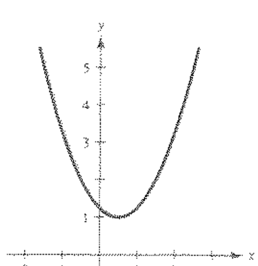
Intercept: $(-1, 0)$



15. $f(x) = x^2 - x + \frac{5}{4} = \left(x - \frac{1}{2}\right)^2 + 1$

Vertex: $\left(\frac{1}{2}, 1\right)$

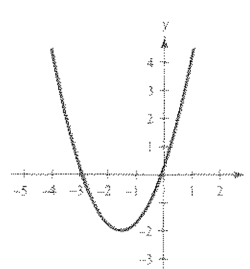
x-intercepts: None



16. $f(x) = x^2 + 3x + \frac{1}{4} = \left(x + \frac{3}{2}\right)^2 - 2$

Vertex: $\left(-\frac{3}{2}, -2\right)$

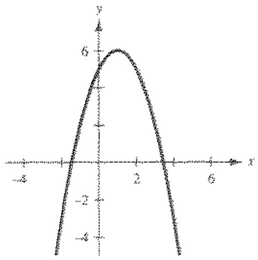
Intercepts: $\left(-\frac{3}{2} \pm \sqrt{2}, 0\right)$



17. $f(x) = -x^2 + 2x + 5 = -(x - 1)^2 + 6$

Vertex: $(1, 6)$

x-intercepts: $\left(1 - \sqrt{6}, 0\right), \left(1 + \sqrt{6}, 0\right)$



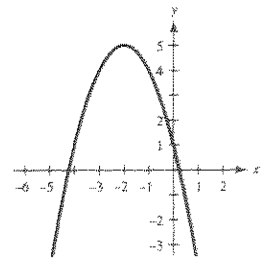
18. $f(x) = -x^2 - 4x + 1 = -1(x^2 + 4x - 1)$

$= -1[(x + 2)^2 - 5]$

$= -(x + 2)^2 + 5$

Vertex: $(-2, 5)$

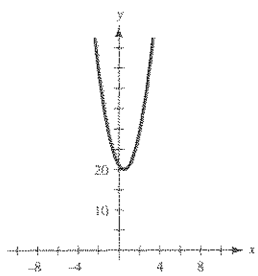
Intercepts: $\left(-2 \pm \sqrt{5}, 0\right)$



19. $h(x) = 4x^2 - 4x + 21 = 4\left(x - \frac{1}{2}\right)^2 + 20$

Vertex: $\left(\frac{1}{2}, 20\right)$

x-intercept: None



20. $f(x) = 2x^2 - x + 1$

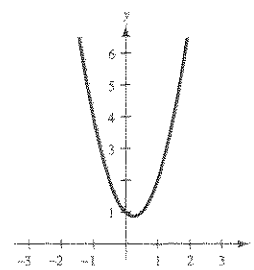
$= 2\left(x^2 - \frac{1}{2}x\right) + 1$

$= 2\left(x - \frac{1}{4}\right)^2 - \frac{1}{8} + 1$

$= 2\left(x - \frac{1}{4}\right)^2 + \frac{7}{8}$

Vertex: $\left(\frac{1}{4}, \frac{7}{8}\right)$

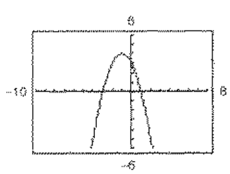
No x-intercepts



21. $f(x) = -(x^2 + 2x - 3) = -(x + 1)^2 + 4$

Vertex: $(-1, 4)$

x-intercepts: $(-3, 0), (1, 0)$



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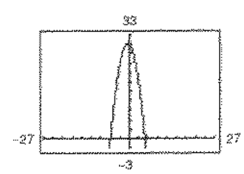


Vertex: $\left(-\frac{1}{2}, \frac{121}{4}\right)$

Intercepts: $(5, 0), (-6, 0)$

$f(x) = -(x^2 + x - 30)$

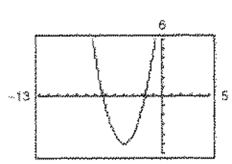
$= -\left(x^2 + x + \frac{1}{4}\right) + \frac{1}{4} + 30$

$= -\left(x + \frac{1}{2}\right)^2 + \frac{121}{4}$

23. $g(x) = x^2 + 8x + 11 = (x + 4)^2 - 5$

Vertex: $(-4, -5)$

x-intercepts: $\left(-4 \pm \sqrt{5}, 0\right)$



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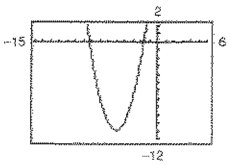


Vertex: $(-5, -11)$

Intercepts: $(-1.683, 0), (-8.317, 0)$

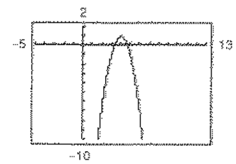
$$f(x) = x^2 + 10x + 14$$
$$= (x^2 + 10x + 25) - 11$$
$$= (x + 5)^2 - 11$$

25. $f(x) = -2x^2 + 16x - 31$

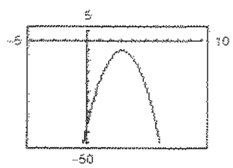
$$= -2\left(x^2 - 8x + \tfrac{31}{2}\right)$$
$$= -2\left(x^2 - 8x + 16 - \tfrac{1}{2}\right)$$
$$= -2(x - 4)^2 + 1$$

Vertex: $(4, 1)$

x-intercept: $\left(4 \pm \frac{1}{2}\sqrt{2}, 0\right)$



26.



Vertex: $(3, -5)$

No x-intercepts

$$f(x) = -4x^2 + 24x - 41$$
$$= -4(x^2 - 6x + 9) + 36 - 41$$
$$= -4(x - 3)^2 - 5$$

27. $(-1, 4)$ is the vertex.

$f(x) = a(x + 1)^2 + 4$

Since the graph passes through the point $(1, 0)$, we have:

$$0 = a(1 + 1)^2 + 4$$
$$0 = 4a + 4$$
$$-1 = a$$

Thus, $f(x) = -(x + 1)^2 + 4$. Note that $(-3, 0)$ is on the parabola.

28. $(-2, -1)$ is the vertex.

$f(x) = a(x + 2)^2 - 1$

Since the graph passes through $(0, 3)$, we have:

$$3 = a(0 + 2)^2 - 1$$
$$3 = 4a - 1$$
$$4 = 4a$$
$$1 = a$$

Thus, $y = (x + 2)^2 - 1$.

29. $(-2, 5)$ is the vertex.

$f(x) = a(x + 2)^2 + 5$

Since the graph passes through the point $(0, 9)$, we have:

$$9 = a(0 + 2)^2 + 5$$
$$4 = 4a$$
$$1 = a$$

$f(x) = 1(x + 2)^2 + 5 = (x + 2)^2 + 5$

30. $(4, 1)$ is the vertex.

$f(x) = a(x - 4)^2 + 1$

Since the graph passes through the point $(6, -7)$, we have:

$$-7 = a(6 - 4)^2 + 1$$
$$-7 = 4a + 1$$
$$-8 = 4a$$
$$-2 = a$$

$f(x) = -2(x - 4)^2 + 1$

31. $(1, -2)$ is the vertex.

$f(x) = a(x - 1)^2 - 2$

Since the graph passes through the point $(-1, 14)$, we have:

$$14 = a(-1 - 1)^2 - 2$$
$$14 = 4a - 2$$
$$16 = 4a$$
$$4 = a$$

$f(x) = 4(x - 1)^2 - 2$

32. $(-4, -1)$ is the vertex.

$f(x) = a(x + 4)^2 - 1$

Since the graph passes through the point $(-2, 4)$, we have:

$4 = a(-2 + 4)^2 - 1$

$5 = 4a$

$a = \frac{5}{4}$

$f(x) = \frac{5}{4}(x + 4)^2 - 1$

33. $\left(\frac{1}{2}, 1\right)$ is the vertex.

$f(x) = a\left(x - \frac{1}{2}\right)^2 + 1$

Since the graph passes through the point $\left(-2, -\frac{21}{5}\right)$, we have:

$-\frac{21}{5} = a\left(-2 - \frac{1}{2}\right)^2 + 1$

$-\frac{21}{5} = \frac{25}{4}a + 1$

$-\frac{26}{5} = \frac{25}{4}a$

$-\frac{104}{125} = a$

$f(x) = -\frac{104}{125}\left(x - \frac{1}{2}\right)^2 + 1$

34. $\left(-\frac{1}{4}, -1\right)$ is the vertex.

$f(x) = a\left(x + \frac{1}{4}\right)^2 - 1$

Since the graph passes through the point $\left(-1, \frac{-17}{16}\right)$, we have:

$-\frac{17}{16} = a\left(0 + \frac{1}{4}\right)^2 - 1$

$-\frac{17}{16} = \frac{1}{16}a - 1$

$-\frac{1}{16} = \frac{1}{16}a$

$a = -1$

$f(x) = -\left(x + \frac{1}{4}\right)^2 - 1$

35. $y = x^2 - 4x - 5$

x-intercepts: $(5, 0), (-1, 0)$

$0 = x^2 - 4x - 5$

$0 = (x - 5)(x + 1)$

$x = 5$ or $x = -1$

36. $y = 2x^2 + 5x - 3$

x-intercepts: $\left(\frac{1}{2}, 0\right), (-3, 0)$

$0 = 2x^2 + 5x - 3$

$0 = (2x - 1)(x + 3)$

$x = \frac{1}{2}, -3$

37. $y = x^2 + 8x + 16$

x-intercept: $(-4, 0)$

$0 = x^2 + 8x + 16$

$0 = (x + 4)^2$

$x = -4$

38. $y = x^2 - 6x + 9$

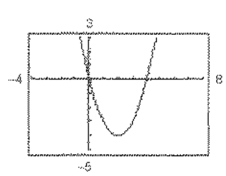
x-intercept: $(3, 0)$

$0 = x^2 - 6x + 9$

$0 = (x - 3)^2$

$x = 3$

39. $y = x^2 - 4x$



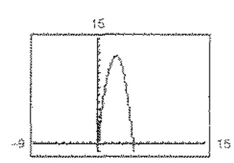
$0 = x^2 - 4x$

$0 = x(x - 4)$

$x = 0$ or $x = 4$

x-intercepts: $(0, 0), (4, 0)$

40. $y = -2x^2 + 10x$



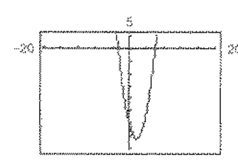
x-intercepts: $(0, 0), (5, 0)$

$0 = -2x^2 + 10x$

$0 = x(-2x + 10)$

$x = 0, x = 5$

41. $y = 2x^2 - 7x - 30$



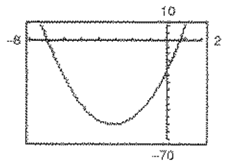
$0 = 2x^2 - 7x - 30$

$0 = (2x + 5)(x - 6)$

$x = -\frac{5}{2}$ or $x = 6$

x-intercepts: $\left(-\frac{5}{2}, 0\right), (6, 0)$

42. $y = 4x^2 + 25x - 21$



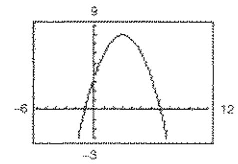
x-intercepts: $(-7, 0), (0.75, 0)$

$0 = 4x^2 + 25x - 21$

$= (x + 7)(4x - 3)$

$x = -7, \frac{3}{4}$

43. $y = -\frac{1}{2}(x^2 - 6x - 7)$



$0 = -\frac{1}{2}(x^2 - 6x - 7)$

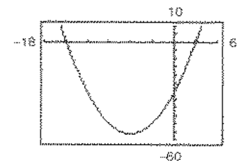
$0 = x^2 - 6x - 7$

$0 = (x + 1)(x - 7)$

$x = -1, 7$

x-intercepts: $(-1, 0), (7, 0)$

44. $y = \frac{7}{10}(x^2 + 12x - 45)$



x-intercepts: $(3, 0), (-15, 0)$

$0 = \frac{7}{10}(x^2 + 12x - 45)$

$0 = x^2 + 12x - 45$

$= (x - 3)(x + 15)$

$x = 3, -15$

45. $f(x) = [x - (-1)](x - 3)$, opens upward

$= (x + 1)(x - 3)$

$= x^2 - 2x - 3$

$g(x) = -[x - (-1)](x - 3)$, opens downward

$= -(x + 1)(x - 3)$

$= -(x^2 - 2x - 3)$

$= -x^2 + 2x + 3$

Note: $f(x) = a(x + 1)(x - 3)$ has x-intercepts $(-1, 0)$ and $(3, 0)$ for all real numbers $a \neq 0$.

46. $f(x) = a(x - 0)(x - 10) = ax(x - 10)$

Many correct answers.

$f(x) = x(x - 10) = x^2 - 10x$ opens upward.

$f(x) = -x(x - 10) = -x^2 + 10x$ opens downward.

47. $f(x) = [x - (-3)]\left[x - \left(-\frac{1}{2}\right)\right](2)$, opens upward

$= (x + 3)\left(x + \frac{1}{2}\right)(2)$

$= (x + 3)(2x + 1)$

$= 2x^2 + 7x + 3$

$g(x) = -(2x^2 + 7x + 3)$, opens downward

$= -2x^2 - 7x - 3$

Note: $f(x) = a(x + 3)(2x + 1)$ has x-intercepts $(-3, 0)$ and $\left(-\frac{1}{2}, 0\right)$ for all real numbers $a \neq 0$.

48. $f(x) = 2\left[x - \left(-\frac{5}{2}\right)\right](x - 2)$

$= 2\left(x + \frac{5}{2}\right)(x - 2)$

$= 2x^2 + x - 10$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -2x^2 - x + 10$

Many other answers possible.

49. Let x = the first number and y = the second number. Then the sum is

$x + y = 110 \implies y = 110 - x.$

The product is

$P(x) = xy = x(110 - x) = 110x - x^2.$

$P(x) = -x^2 + 110x$

$= -(x^2 - 110x + 3025 - 3025)$

$= -[(x - 55)^2 - 3025]$

$= -(x - 55)^2 + 3025$

The maximum value of the product occurs at the vertex of $P(x)$ and is 3025. This happens when $x = y = 55$.

50. Let x = first number and y = second number. Then, $x + y = S$, $y = S - x$. The product is

$P(x) = xy = x(S - x)$.

$$\begin{aligned}P(x) &= Sx - x^2\\ &= -x^2 + Sx\\ &= -\left(x^2 - Sx + \frac{S^2}{4} - \frac{S^2}{4}\right)\\ &= -\left(x - \frac{S}{2}\right)^2 + \frac{S^2}{4}\end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is $S^2/4$. This happens when $x = y = S/2$.

51. Let x be the first number and y be the second number. Then $x + 2y = 24 \implies x = 24 - 2y$. The product is $P = xy = (24 - 2y)y = 24y - 2y^2$.

Completing the square,

$$\begin{aligned}P &= -2y^2 + 24y\\ &= -2(y^2 - 12y + 36) + 72\\ &= -2(y - 6)^2 + 72.\end{aligned}$$

The maximum value of the product P occurs at the vertex of the parabola and equals 72. This happens when $y = 6$ and $x = 24 - 2(6) = 12$.

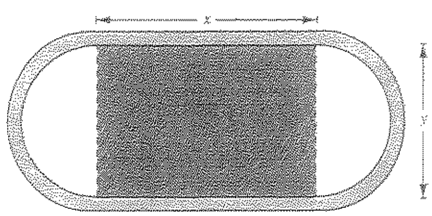
52. Let x = first number and y = second number. Then $x + 3y = 42$, $y = \frac{1}{3}(42 - x)$. The product is

$P(x) = xy = x\frac{1}{3}(42 - x) = 14x - \frac{1}{3}x^2$.

$$\begin{aligned}P(x) &= -\tfrac{1}{3}x^2 + 14x\\ &= -\tfrac{1}{3}(x^2 - 42x)\\ &= -\tfrac{1}{3}(x^2 - 42x + 441) + 147\\ &= -\tfrac{1}{3}(x - 21)^2 + 147.\end{aligned}$$

The maximum value of the product is 147, and occurs when $x = 21$ and $y = \frac{1}{3}(42 - 21) = 7$.

53. (a)



(b) Radius of semicircular ends of track: $r = \frac{1}{2}y$

Distance around two semicircular parts of track:

$$d = 2\pi r = 2\pi\left(\frac{1}{2}y\right) = \pi y$$

(c) Distance traveled around track in one lap:

$$\begin{aligned}d &= \pi y + 2x = 200\\ \pi y &= 200 - 2x\\ y &= \frac{200 - 2x}{\pi}\end{aligned}$$

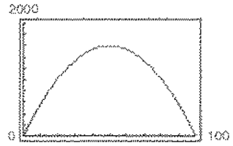
(d) Area of rectangular region:

$$\begin{aligned}A = xy &= x\left(\frac{200 - 2x}{\pi}\right)\\ &= \frac{1}{\pi}(200x - 2x^2)\\ &= -\frac{2}{\pi}(x^2 - 100x)\\ &= -\frac{2}{\pi}(x^2 - 100x + 2500 - 2500)\\ &= -\frac{2}{\pi}(x - 50)^2 + \frac{5000}{\pi}\end{aligned}$$

The area is maximum when $x = 50$ and

$$y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi}.$$

(e)



The area is maximum when $x = 50$ and

$$y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi}.$$

54. (a) $4x + 3y = 200 \implies y = \frac{1}{3}(200 - 4x) \implies A = 2xy = 2x\frac{1}{3}(200 - 4x) = \frac{8x}{3}(50 - x)$

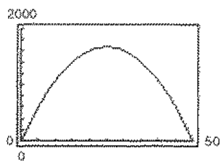
(b)

x	y	Area
2	$\frac{1}{3}[200 - 4(2)]$	$2xy = 256$
4	$\frac{1}{3}[200 - 4(4)]$	$2xy \approx 491$
6	$\frac{1}{3}[200 - 4(6)]$	$2xy = 704$
8	$\frac{1}{3}[200 - 4(8)]$	$2xy = 896$
10	$\frac{1}{3}[200 - 4(10)]$	$2xy \approx 1067$
12	$\frac{1}{3}[200 - 4(12)]$	$2xy = 1216$

x	y	Area
20	$\frac{1}{3}[200 - 4(20)]$	$2xy = 1600$
22	$\frac{1}{3}[200 - 4(22)]$	$2xy \approx 1643$
24	$\frac{1}{3}[200 - 4(24)]$	$2xy = 1664$
26	$\frac{1}{3}[200 - 4(26)]$	$2xy = 1664$
28	$\frac{1}{3}[200 - 4(28)]$	$2xy \approx 1643$
30	$\frac{1}{3}[200 - 4(30)]$	$2xy = 1600$

Maximum area if $x = 25$, $y = 33\frac{1}{3}$

(c) $A = \frac{8x(50 - x)}{3}$



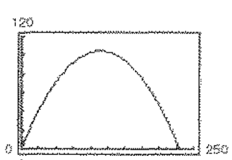
Maximum if $x = 25$, $y = 33\frac{1}{3}$

(d) $$\begin{aligned} A &= \frac{8}{3}x(50 - x) \\ &= -\frac{8}{3}(x^2 - 50x) \\ &= -\frac{8}{3}(x^2 - 50x + 625 - 625) \\ &= -\frac{8}{3}[(x - 25)^2 - 625] \\ &= -\frac{8}{3}(x - 25)^2 + \frac{5000}{3} \end{aligned}$$

The maximum area occurs at the vertex and is $5000/3$ square feet. This happens when $x = 25$ feet and $y = (200 - 4(25))/3 = 100/3$ feet. The dimensions are $2x = 50$ feet by $33\frac{1}{3}$ feet.

(e) The results are the same.

55. (a)



(b) When $x = 0$, $y = \frac{3}{2}$ feet.

(c) The vertex occurs at

$$x = \frac{-b}{2a} = \frac{-9/5}{2(-16/2025)} = \frac{3645}{32} \approx 113.9.$$

The maximum height is

$$y = \frac{-16}{2025}\left(\frac{3645}{32}\right)^2 + \frac{9}{5}\left(\frac{3645}{32}\right) + \frac{3}{2}$$

$$\approx 104.0 \text{ feet.}$$

(d) Using a graphing utility, the zero of y occurs at $x \approx 228.6$, or 228.6 feet from the punter.

56. $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$

The maximum height of the dive occurs at the vertex, $x = \frac{-b}{2a} = -\frac{24/9}{2(-4/9)} = 3.$

The height at $x = 3$ is

$$-\frac{4}{9}(3)^2 + \frac{24}{9}(3) + 12 = 16.$$

The maximum height of the dive is 16 feet.

57. $C = 800 - 10x + 0.25x^2$

x	10	15	20	25	30
C	725	706.25	700	706.25	725

From the table, the minimum cost seems to be at $x = 20$.

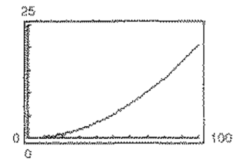
The minimum cost occurs at the vertex.

$$x = \frac{-b}{2a} = -\frac{(-10)}{2(0.25)} = \frac{10}{0.5} = 20$$

$C(20) = 700$ is the minimum cost.

Graphically, you could graph $C = 800 - 10x + 0.25x^2$ in the window $[0, 40] \times [0, 1000]$ and find the vertex $(20, 700)$.

58. (a)



$0.002s^2 + 0.05s - 0.029$

(b) The parabola intersects $y = 10$ at $s \approx 59.4$. Thus, the maximum speed is 59.4 mph. Analytically,

$$0.002s^2 + 0.05s - 0.029 = 10$$
$$2s^2 + 50s - 29 = 10{,}000$$
$$2s^2 + 50s - 10{,}029 = 0.$$

Using the Quadratic Formula,

$$s = \frac{-50 \pm \sqrt{50^2 - 4(2)(-10{,}029)}}{2(2)}$$
$$= \frac{-50 \pm \sqrt{82{,}732}}{4} \approx -84.4, 59.4.$$

The maximum speed is the positive root, 59.4 mph.

59. (a) $R(20) = -25(20)^2 + 1200(20)$

$= \$14{,}000$ thousand

$R(25) = -25(25)^2 + 1200(25)$

$= \$14{,}375$ thousand

$R(30) = -25(30)^2 + 1200(30)$

$= \$13{,}500$ thousand

(b) The vertex occurs at

$$p = \frac{-b}{2a} = \frac{-1200}{2(-25)} = \$24.$$

(c) $R(24) = -25(24)^2 + 1200(24)$

$= \$14{,}400$ thousand

(d) Answers will vary.

60. (a) $R(4) = -12(4)^2 + 150(4) = \$408$

$R(6) = -12(6)^2 + 150(6) = \$468$

$R(8) = -12(8)^2 + 150(8) = \$432$

(b) The vertex occurs at

$$p = \frac{-b}{2a} = \frac{-150}{2(-12)} = \frac{25}{4} = 6.25.$$

The price is \$6.25 per pet.

(c) The maximum revenue is

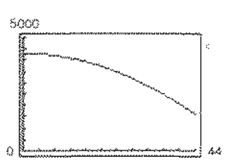
$$R\left(\frac{25}{4}\right) = -12\left(\frac{25}{4}\right)^2 + 150\left(\frac{25}{4}\right) = \$468.75.$$

(d) Answers will vary.

61. $C(t) = 4306 - 3.4t - 1.32t^2, 0 \le t \le 44$

($t = 0$ corresponds to 1960.)

(a)



(b) The maximum consumption per year of 4306 cigarettes per person per year occurred in 1960 ($t = 0$). Answers will vary.

(c) For 2000, $C(40) = 2058$.

$$(2058)\frac{209{,}117{,}000}{48{,}306{,}000} \approx 8909 \text{ cigarettes per smoker per year,}$$

$$\frac{8909}{365} \approx 24 \text{ cigarettes per smoker per day}$$

62. $S = -28.40t^2 + 218.1t + 2435,\ 0 \le t \le 14$

(a) The vertex is $\dfrac{-b}{2a} = \dfrac{-218.1}{2(-28.4)} \approx 3.8$, or 1993.

(b) For 2004, $t = 14$ and $S \approx -78$, or $-\$78{,}000{,}000$. Clearly the model is not accurate past 2003.

(c) Probably not. Answers will vary.

63. True

$$-12x^2 - 1 = 0$$

$$12x^2 = -1, \text{ impossible}$$

64. True. For $f(x)$, $\dfrac{-b}{2a} = -\dfrac{-10}{2(-4)} = -\dfrac{10}{8} = -\dfrac{5}{4}$.

For $g(x)$, $\dfrac{-b}{2a} = \dfrac{-30}{2(12)} = \dfrac{-30}{24} = \dfrac{-5}{4}$.

In both cases, $x = -\frac{5}{4}$ is the axis of symmetry.

65. The parabola opens downward and the vertex is $(-2, -4)$. Matches (c) and (d).

66. The parabola opens upward and the vertex is $(1, 3)$. Matches (a).

67. For $a < 0$, $f(x) = a\left(x + \dfrac{b}{2a}\right)^2 + \left(c - \dfrac{b^2}{4a}\right)$ is a maximum when $x = \dfrac{-b}{2a}$. In this case, the maximum value is $c - \dfrac{b^2}{4a}$. Hence,

$$25 = -75 - \frac{b^2}{4(-1)}$$

$$-100 = 300 - b^2$$

$$400 = b^2$$

$$b = \pm 20.$$

68. For $a < 0$, $f(x) = a\left(x + \dfrac{b}{2a}\right)^2 + \left(c - \dfrac{b^2}{4a}\right)$ is a maximum when $x = \dfrac{-b}{2a}$. In this case, the maximum value is $c - \dfrac{b^2}{4a}$. Hence,

$$48 = -16 - \frac{b^2}{4(-1)}$$

$$-192 = 64 - b^2$$

$$b^2 = 256$$

$$b = \pm 16.$$

69. For $a > 0$, $f(x) = a\left(x + \dfrac{b}{2a}\right)^2 + \left(c - \dfrac{b^2}{4a}\right)$ is a minimum when $x = \dfrac{-b}{2a}$. In this case, the minimum value is $c - \dfrac{b^2}{4a}$. Hence,

$$10 = 26 - \frac{b^2}{4}$$

$$40 = 104 - b^2$$

$$b^2 = 64$$

$$b = \pm 8.$$

70. For $a > 0$, $f(x) = a\left(x + \dfrac{b}{2a}\right)^2 + \left(c - \dfrac{b^2}{4a}\right)$ is a minimum when $x = \dfrac{-b}{2a}$. In this case, the minimum value is $c - \dfrac{b^2}{4a}$. Hence,

$$-50 = -25 - \frac{b^2}{4}$$

$$-200 = -100 - b^2$$

$$b^2 = 100$$

$$b = \pm 10.$$

71. Model (a) is preferable. $a > 0$ means the parabola opens upward and profits are increasing for t to the right of the vertex,

$$t \geq -\frac{b}{(2a)}.$$

72. $y = ax^2 + bx - 4$

$(1, 0)$ on graph: $0 = a + b - 4$

$(4, 0)$ on graph: $0 = 16a + 4b - 4$

From the first equation, $b = 4 - a$.

Thus, $0 = 16a + 4(4 - a) - 4 = 12a + 12 \implies a = -1$ and hence $b = 5$, and $y = -x^2 + 5x - 4$.

73. $x + y = 8 \implies y = 8 - x$

Then $-\frac{2}{3}x + y = -\frac{2}{3}x + (8 - x) = 6 \implies -\frac{5}{3}x = -2 \implies x = \frac{6}{5}$ and $y = 8 - \frac{6}{5} = \frac{34}{5}$.

$(1.2, 6.8)$

74. $y = 3x - 10 = \frac{1}{4}x + 1$

$$12x - 40 = x + 4$$

$$11x = 44$$

$$x = 4$$

The graphs intersect at $(4, 2)$.

75. $y = x + 3 = 9 - x^2$

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

$$x = -3, x = 2$$

Thus, $(-3, 0)$ and $(2, 5)$ are the points of intersection.

76. $y = x^3 + 2x - 1 = -2x + 15$

$$x^3 + 4x - 16 = 0$$

$$(x - 2)(x^2 + 2x + 8) = 0$$

$$x = 2$$

The graphs intersect at $(2, 11)$.

77. Answers will vary. (Make a Decision)

Section 2.2 Polynomial Functions of Higher Degree

- You should know the following basic principles about polynomials.
- $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$, $a_n \neq 0$, is a polynomial function of degree n.
- If f is of odd degree and
  (a) $a_n > 0$, then
    1. $f(x) \to \infty$ as $x \to \infty$.
    2. $f(x) \to -\infty$ as $x \to -\infty$.

  (b) $a_n < 0$, then
    1. $f(x) \to -\infty$ as $x \to \infty$.
    2. $f(x) \to \infty$ as $x \to -\infty$.
- If f is of even degree and
  (a) $a_n > 0$, then
    1. $f(x) \to \infty$ as $x \to \infty$.
    2. $f(x) \to \infty$ as $x \to -\infty$.

  (b) $a_n < 0$, then
    1. $f(x) \to -\infty$ as $x \to \infty$.
    2. $f(x) \to -\infty$ as $x \to -\infty$.
- The following are equivalent for a polynomial function.
  (a) $x = a$ is a zero of a function.
  (b) $x = a$ is a solution of the polynomial equation $f(x) = 0$.
  (c) $(x - a)$ is a factor of the polynomial.
  (d) $(a, 0)$ is an x-intercept of the graph of f.
- A polynomial of degree n has at most n distinct zeros.
- If f is a polynomial function such that $a < b$ and $f(a) \neq f(b)$, then f takes on every value between $f(a)$ and $f(b)$ in the interval $[a, b]$.
- If you can find a value where a polynomial is positive and another value where it is negative, then there is at least one real zero between the values.

Vocabulary Check

1. continuous
2. Leading Coefficient Test
3. n, $n - 1$, relative extrema
4. solution, $(x - a)$, x-intercept
5. touches, crosses
6. Intermediate Value

1. $f(x) = -2x + 3$ is a line with y-intercept $(0, 3)$. Matches graph (f).

2. $f(x) = x^2 - 4x$ is a parabola with intercepts $(0, 0)$ and $(4, 0)$ and opens upward. Matches graph (h).

3. $f(x) = -2x^2 - 5x$ is a parabola with x-intercepts $(0, 0)$ and $\left(-\frac{5}{2}, 0\right)$ and opens downward. Matches graph (c).

4. $f(x) = 2x^3 - 3x + 1$ has intercepts $(0, 1)$, $(1, 0)$, $\left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}, 0\right)$ and $\left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}, 0\right)$. Matches graph (a).

5. $f(x) = -\frac{1}{4}x^4 + 3x^2$ has intercepts $(0, 0)$ and $\left(\pm 2\sqrt{3}, 0\right)$. Matches graph (e).

6. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$ has y-intercept $\left(0, -\frac{4}{3}\right)$. Matches graph (d).

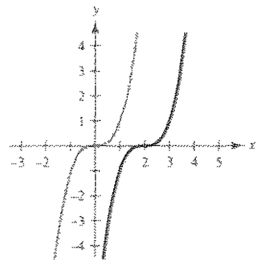
7. $f(x) = x^4 + 2x^3$ has intercepts $(0, 0)$ and $(-2, 0)$. Matches graph (g).

8. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ has intercepts $(0, 0)$, $(1, 0)$, $(-1, 0)$, $(3, 0)$, $(-3, 0)$. Matches (b).

9. $y = x^3$

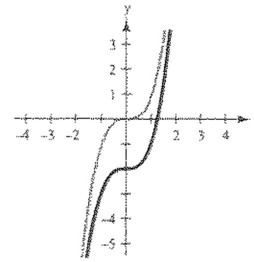
(a) $f(x) = (x - 2)^3$

Horizontal shift two units to the right



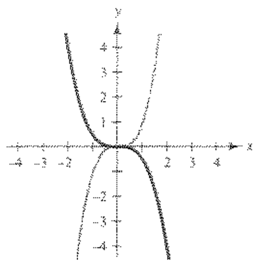
(b) $f(x) = x^3 - 2$

Vertical shift two units downward



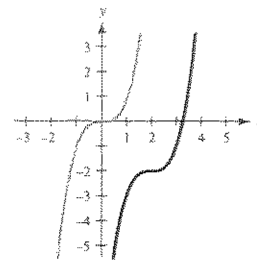
(c) $f(x) = -\frac{1}{2}x^3$

Reflection in the x-axis and a vertical shrink



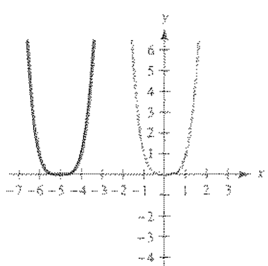
(d) $f(x) = (x - 2)^3 - 2$

Horizontal shift two units to the right and a vertical shift two units downward



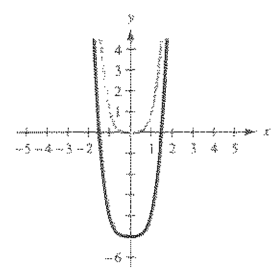
10. $y = x^4$

(a) $f(x) = (x + 5)^4$



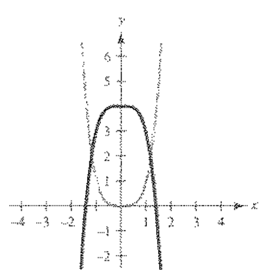
Horizontal shift five units to the left

(b) $f(x) = x^4 - 5$



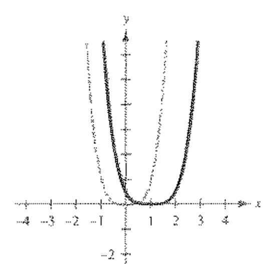
Vertical shift five units downward

(c) $f(x) = 4 - x^4$



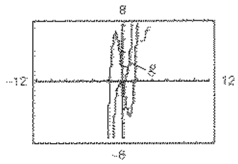
Reflection in the x-axis and then a vertical shift four units upward

(d) $f(x) = \frac{1}{2}(x - 1)^4$

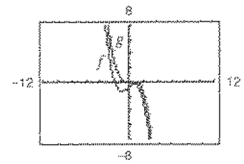


Horizontal shift one unit to the right and a vertical shrink

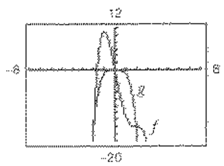
11. $f(x) = 3x^3 - 9x + 1;\ g(x) = 3x^3$



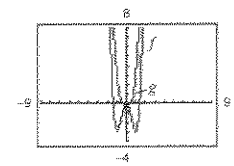
12. $f(x) = -\frac{1}{3}(x^3 - 3x + 2),\ g(x) = -\frac{1}{3}x^3$



13. $f(x) = -(x^4 - 4x^3 + 16x);\ g(x) = -x^4$



14. $f(x) = 3x^4 - 6x^2,\ g(x) = 3x^4$



15. $f(x) = 2x^4 - 3x + 1$

Degree: 4

Leading coefficient: 2

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

16. $h(x) = 1 - x^6$

Degree: 6

Leading coefficient: -1

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

17. $g(x) = 5 - \frac{7}{2}x - 3x^2$

Degree: 2

Leading coefficient: -3

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

18. $f(x) = \frac{1}{3}x^3 + 5x$

Degree: 3

Leading coefficient: $\frac{1}{3}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

19. Degree: 5 (odd)

Leading coefficient: $\frac{6}{3} = 2 > 0$

Falls to the left and rises to the right

20. Degree: 7 (odd)

Leading coefficient: $\frac{3}{4} > 0$

Falls to the left and rises to the right

21. $h(t) = -\frac{2}{3}(t^2 - 5t + 3)$

Degree: 2

Leading coefficient: $-\frac{2}{3}$

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

22. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$

Degree: 3

Leading coefficient: $-\frac{7}{8}$

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

23. $f(x) = x^2 - 25$
$= (x + 5)(x - 5)$
$x = \pm 5$

24. $f(x) = 49 - x^2$
$= (7 - x)(7 + x)$
$x = \pm 7$

25. $h(t) = t^2 - 6t + 9$
$= (t - 3)^2$
$t = 3$ (multiplicity 2)

26. $f(x) = x^2 + 10x + 25$
$= (x + 5)^2$
$x = -5$ (multiplicity 2)

27. $f(x) = x^2 + x - 2$
$= (x + 2)(x - 1)$
$x = -2, 1$

28. $f(x) = 2x^2 - 14x + 24$
$= 2(x^2 - 7x + 12)$
$= 2(x - 3)(x - 4)$
$x = 3, 4$

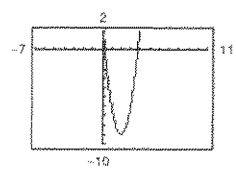
29. $f(t) = t^3 - 4t^2 + 4t$
$= t(t - 2)^2$
$t = 0, 2$ (multiplicity 2)

30. $f(x) = x^4 - x^3 - 20x^2$
$= x^2(x^2 - x - 20)$
$= x^2(x + 4)(x - 5)$
$x = -4, 5, 0$ (multiplicity 2)

31. $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2}$
$= \frac{1}{2}(x^2 + 5x - 3)$
$x = \frac{-5 \pm \sqrt{25 - 4(-3)}}{2} = -\frac{5}{2} \pm \frac{\sqrt{37}}{2}$
$\approx 0.5414, -5.5414$

32. $f(x) = \frac{5}{3}x^2 + \frac{8}{3}x - \frac{4}{3}$
$= \frac{1}{3}(5x^2 + 8x - 4)$
$= \frac{1}{3}(5x - 2)(x + 2)$
$x = \frac{2}{5}, -2$

33. (a)

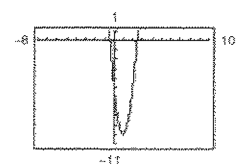


(b) $x \approx 3.732, 0.268$

(c) $f(x) = 3x^2 - 12x + 3$
$= 3(x^2 - 4x + 1)$
$x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$

34. $g(x) = 5x^2 - 10x - 5$

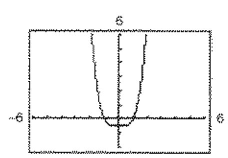
(a)



(b) Zeros: $-0.414, 2.414$

(c) $g(x) = 5(x^2 - 2x - 1)$
$x = \frac{2 \pm \sqrt{4 - 4(-1)}}{2} = 1 \pm \sqrt{2}$
$(\approx -0.414, 2.414)$
$(1 \pm \sqrt{2}, 0)$

35. (a)

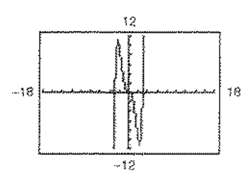


(b) $t = \pm 1$

(c) $g(t) = \frac{1}{2}t^4 - \frac{1}{2}$
$= \frac{1}{2}(t + 1)(t - 1)(t^2 + 1)$
$t = \pm 1$

36. $y = \frac{1}{4}x^3(x^2 - 9)$

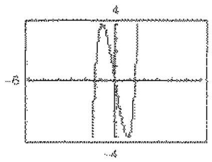
(a)



(b) Zeros: $0, \pm 3$

(c) $0 = \frac{1}{4}x^3(x^2 - 9)$
$x = 0, \pm 3$
x-intercepts: $(0, 0), (\pm 3, 0)$

37. (a)



(b) $x = 0, 1.414, -1.414$

(c) $f(x) = x^5 + x^3 - 6x$

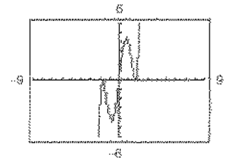
$= x(x^4 + x^2 - 6)$

$= x(x^2 + 3)(x^2 - 2)$

$x = 0, \pm\sqrt{2}$

38. $g(t) = t^5 - 6t^3 + 9t$

(a)



(b) Zeros: $0, \pm 1.732$

(c) $g(t) = t^5 - 6t^3 + 9t$

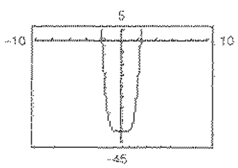
$= t(t^4 - 6t^2 + 9)$

$= t(t^2 - 3)^2$

$t = 0, \pm\sqrt{3} \quad (\approx 0, \pm 1.732)$

$(0, 0), (\pm\sqrt{3}, 0)$

39. (a)



(b) $2.236, -2.236$

(c) $f(x) = 2x^4 - 2x^2 - 40$

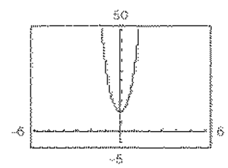
$= 2(x^4 - x^2 - 20)$

$= 2(x^2 + 4)(x + \sqrt{5})(x - \sqrt{5})$

$x = \pm\sqrt{5}$

40. $f(x) = 5x^4 + 15x^2 + 10$

(a)



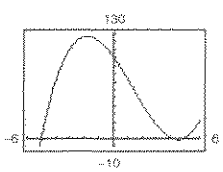
(b) No real zeros

(c) $f(x) = 5(x^4 + 3x^2 + 2)$

$= 5(x^2 + 1)(x^2 + 2) > 0$

No real zeros

41. (a)



(b) $x = 4, 5, -5$

(c) $f(x) = x^3 - 4x^2 - 25x + 100$

$= x^2(x - 4) - 25(x - 4)$

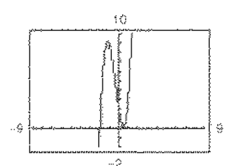
$= (x^2 - 25)(x - 4)$

$= (x - 5)(x + 5)(x - 4)$

$x = \pm 5, 4$

42. $y = 4x^3 + 4x^2 - 7x + 2$

(a)



(b) Zeros: $-2, \frac{1}{2}$

(c) $0 = 4x^3 + 4x^2 - 7x + 2$

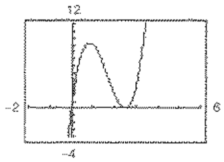
$= (2x - 1)(2x^2 + 3x - 2)$

$= (2x - 1)(2x - 1)(x + 2)$

$x = -2, \frac{1}{2}$

x-intercepts: $(-2, 0), \left(\frac{1}{2}, 0\right)$

43. (a)



(b) $x = 0, \frac{5}{2}$

(c) $y = 4x^3 - 20x^2 + 25x$

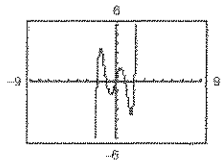
$0 = 4x^3 - 20x^2 + 25x$

$0 = x(2x - 5)^2$

$x = 0$ or $x = \frac{5}{2}$ (multiplicity 2)

44. $y = x^5 - 5x^3 + 4x$

(a)



(b) Zeros: $0, \pm 1, \pm 2$

(c) $y = x^5 - 5x^3 + 4x$

$= x(x^4 - 5x^2 + 4)$

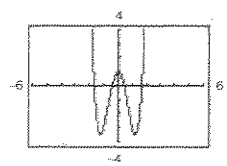
$= x(x^2 - 4)(x^2 - 1)$

$= x(x - 2)(x + 2)(x - 1)(x + 1)$

Zeros: $0, \pm 1, \pm 2$

$(0, 0), (\pm 1, 0), (\pm 2, 0)$

45. $f(x) = 2x^4 - 6x^2 + 1$

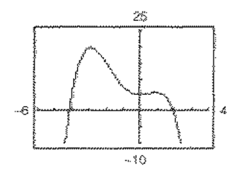


Zeros: $x \approx \pm 0.421, \pm 1.680$

Relative maximum: $(0, 1)$

Relative minimums:
$(1.225, -3.5), (-1.225, -3.5)$

46. $f(x) = -\frac{3}{8}x^4 - x^3 + 2x^2 + 5$

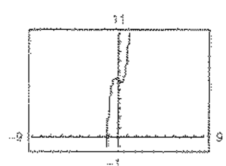


Real zeros: $-4.142, 1.934$

Relative maximums: $(0.915, 5.646)$, $(-2.915, 19.688)$

Relative minimum: $(0, 5)$

47. $f(x) = x^5 + 3x^3 - x + 6$

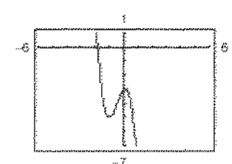


Zeros: $x \approx -1.178$

Relative maximum: $(-0.324, 6.218)$

Relative minimum: $(0.324, 5.782)$

48. $f(x) = -3x^3 - 4x^2 + x - 3$



Real zero: -1.819

Relative maximum: $(0.111, -2.942)$

Relative minimum: $(-1, -5)$

49. $f(x) = (x - 0)(x - 4) = x^2 - 4x$

Note: $f(x) = a(x - 0)(x - 4) = ax(x - 4)$ has zeros 0 and 4 for all nonzero real numbers a.

50. $f(x) = (x + 7)(x - 2) = x^2 + 5x - 14$

51. $f(x) = (x - 0)(x + 2)(x + 3) = x^3 + 5x^2 + 6x$

Note: $f(x) = ax(x + 2)(x + 3)$ has zeros $0, -2$, and -3 for all nonzero real numbers a.

52. $f(x) = (x - 0)(x - 2)(x - 5) = x^3 - 7x^2 + 10x$

53. $f(x) = (x - 4)(x + 3)(x - 3)(x - 0)$

$= (x - 4)(x^2 - 9)x$

$= x^4 - 4x^3 - 9x^2 + 36x$

Note: $f(x) = a(x^4 - 4x^3 - 9x^2 + 36x)$ has zeros 4, -3, 3, and 0 for all nonzero real numbers a.

54. $f(x) = (x - (-2))(x - (-1))(x - 0)(x - 1)(x - 2)$

$= x(x + 2)(x + 1)(x - 1)(x - 2)$

$= x(x^2 - 4)(x^2 - 1)$

$= x(x^4 - 5x^2 + 4)$

$= x^5 - 5x^3 + 4x$

Note: $f(x) = a\,x(x + 2)(x + 1)(x - 1)(x - 2)$ has zeros $-2, -1, 0, 1, 2$ for all nonzero real numbers a.

55. $f(x) = \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right]$

$= \left[(x - 1) - \sqrt{3}\right]\left[(x - 1) + \sqrt{3}\right]$

$= (x - 1)^2 - \left(\sqrt{3}\right)^2$

$= x^2 - 2x + 1 - 3$

$= x^2 - 2x - 2$

Note: $f(x) = a(x^2 - 2x - 2)$ has zeros $1 + \sqrt{3}$ and $1 - \sqrt{3}$ for all nonzero real numbers a.

56. $f(x) = \left(x - \left(6 + \sqrt{3}\right)\right)\left(x - \left(6 - \sqrt{3}\right)\right)$

$= \left((x - 6) - \sqrt{3}\right)\left((x - 6) + \sqrt{3}\right)$

$= (x - 6)^2 - 3$

$= x^2 - 12x + 36 - 3$

$= x^2 - 12x + 33$

Note: $f(x) = a\left(x - \left(6 + \sqrt{3}\right)\right)\left(x - \left(6 - \sqrt{3}\right)\right)$ has zeros $6 + \sqrt{3}$ and $6 - \sqrt{3}$ for all nonzero real numbers a.

57. $f(x) = (x - 2)\left[x - \left(4 + \sqrt{5}\right)\right]\left[x - \left(4 - \sqrt{5}\right)\right]$

$= (x - 2)\left[(x - 4) - \sqrt{5}\right]\left[(x - 4) + \sqrt{5}\right]$

$= (x - 2)[(x - 4)^2 - 5]$

$= x^3 - 10x^2 + 27x - 22$

Note: $f(x) = a(x - 2)[(x - 4)^2 - 5]$ has zeros 2, $4 + \sqrt{5}$, and $4 - \sqrt{5}$ for all nonzero real numbers a.

58. $f(x) = (x - 4)\left(x - \left(2 + \sqrt{7}\right)\right)\left(x - \left(2 - \sqrt{7}\right)\right)$

$= (x - 4)\left((x - 2) - \sqrt{7}\right)\left((x - 2) + \sqrt{7}\right)$

$= (x - 4)\left((x - 2)^2 - 7\right)$

$= (x - 4)(x^2 - 4x - 3)$

$= x^3 - 8x^2 + 13x + 12$

Note: $f(x) = a(x - 4)(x^2 - 4x - 3)$ has zeros 4, $2 \pm \sqrt{7}$ for all nonzero real numbers a.

59. $f(x) = (x + 2)^2(x + 1) = x^3 + 5x^2 + 8x + 4$

Note: $f(x) = a(x + 2)^2(x + 1)$ has zeros $-2, -2$, and -1 for all nonzero real numbers a.

60. $f(x) = (x - 3)(x - 2)^3$

$= x^4 - 9x^3 + 30x^2 - 44x + 24$

Note: $f(x) = a(x - 3)(x - 2)^3$ has zeros 3, 2, 2, 2 for all nonzero real numbers a.

61. $f(x) = (x + 4)^2(x - 3)^2$

$= x^4 + 2x^3 - 23x^2 - 24x + 144$

Note: $f(x) = a(x + 4)^2(x - 3)^2$ has zeros $-4, -4$, 3, 3 for all nonzero real numbers a.

62. $f(x) = (x + 5)^3(x - 0)^2$

$= x^5 + 15x^4 + 75x^3 + 125x^2$

Note: $f(x) = a(x + 5)^3x^2$ has zeros $-5, -5, -5$, 0, 0 for all nonzero real numbers a.

63. $f(x) = -(x + 1)^2(x + 2)$

$= -x^3 - 4x^2 - 5x - 2$

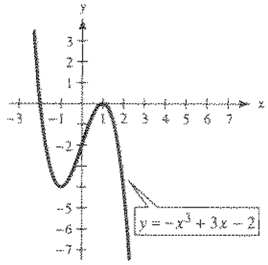
Note: $f(x) = a(x + 1)^2(x + 2)^2$, $a < 0$, has zeros $-1, -1, -2$, rises to the left, and falls to the right.

64. $f(x) = -(x + 1)^2(x - 4)^2$

$= -x^4 + 6x^3 - x^2 - 24x - 16$

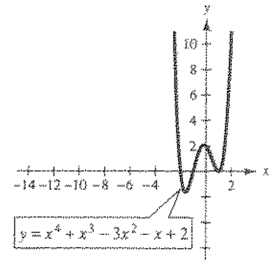
Note: $f(x) = a(x + 1)^2(x - 4)^2$, $a < 0$, has zeros $-1, -1, 4, 4$, falls to the left and falls to the right.

65.



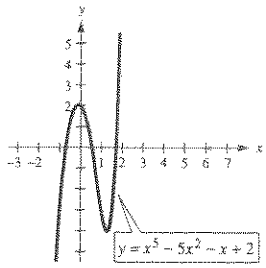
For example,
$f(x) = -(x + 2)(x - 1)^2 = -x^3 + 3x - 2.$

66.

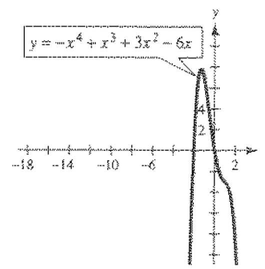


For example, $f(x) = (x + 2)(x + 1)(x - 1)^2$.

67.



68.

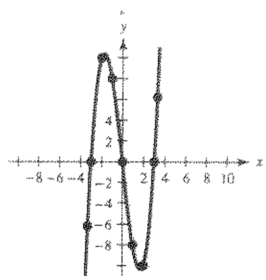


69. (a) The degree of f is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3 - 9x = x(x^2 - 9) = x(x - 3)(x + 3)$

Zeros: $0, 3, -3$

(c) and (d)



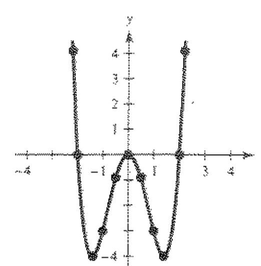
70. (a) The degree of g is even and the leading coefficient is 1. The graph rises to the left and rises to the right.

(b) $g(x) = x^4 - 4x^2 = x^2(x^2 - 4)$

$= x^2(x - 2)(x + 2)$

Zeros: $0, 2, -2$: $(0, 0)$, $(\pm 2, 0)$

(c) and (d)

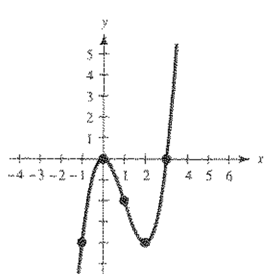


71. (a) The degree of f is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3 - 3x^2 = x^2(x - 3)$

Zeros: $0, 3$

(c) and (d)

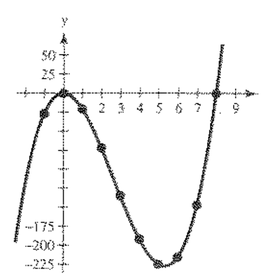


72. (a) The degree of f is odd and the leading coefficient is 3. The graph falls to the left and rises to the right.

(b) $f(x) = 3x^3 - 24x^2 = 3x^2(x - 8)$

Zeros: $0, 8$

(c) and (d)

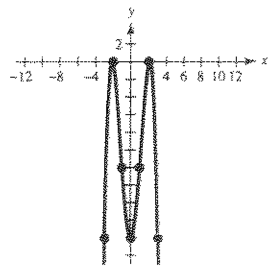


73. (a) The degree of f is even and the leading coefficient is -1. The graph falls to the left and falls to the right.

(b) $f(x) = -x^4 + 9x^2 - 20 = -(x^2 - 4)(x^2 - 5)$

Zeros: $\pm 2, \pm\sqrt{5}$: $(\pm 2, 0)$, $(\pm\sqrt{5}, 0)$

(c) and (d)

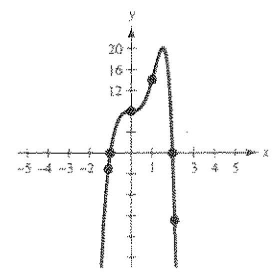


74. (a) The degree is even and the leading coefficient is -1. The graph falls to the left and falls to the right.

(b) $f(x) = -x^6 + 7x^3 + 8 = -(x^3 + 1)(x^3 - 8)$

Zeros: $-1, 2$: $(-1, 0)$, $(2, 0)$

(c) and (d)



75. (a) The degree is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

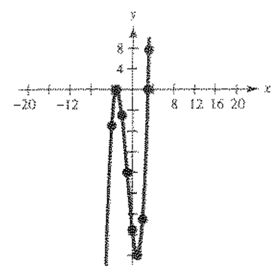
(b) $x^3 + 3x^2 - 9x - 27 = x^2(x + 3) - 9(x + 3)$

$= (x^2 - 9)(x + 3)$

$= (x - 3)(x + 3)^2$

Zeros: $3, -3$: $(3, 0)$, $(-3, 0)$

(c) and (d)



76. (a) The degree is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

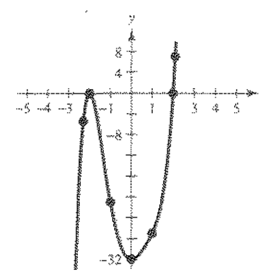
(b) $x^5 - 4x^3 + 8x^2 - 32 = x^3(x^2 - 4) + 8(x^2 - 4)$

$= (x^3 + 8)(x^2 - 4)$

$= (x + 2)(x^2 - 2x + 4)(x - 2)(x + 2)$

Zeros: $-2, 2$: $(\pm 2, 0)$

(c) and (d)



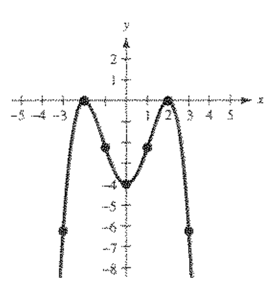
77. $g(t) = -\frac{1}{4}t^4 + 2t^2 - 4$

(a) Falls to left and falls to right; $\left(-\frac{1}{4} < 0\right)$

(b) $g(t) = -\frac{1}{4}(t^4 - 8t^2 + 16) = -\frac{1}{4}(t^2 - 4)^2$

$t = -2, -2, 2, 2 \Longrightarrow (-2, 0), (2, 0)$; zeros

(c) and (d)



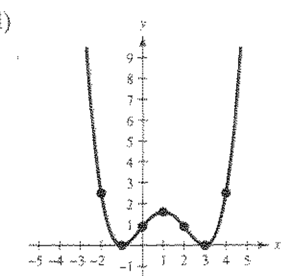
78. $g(x) = \frac{1}{10}(x^4 - 4x^3 - 2x^2 + 12x + 9)$

(a) Rises to right and rises to left; $\left(\frac{1}{10} > 0\right)$

(b) $g(x) = \frac{1}{10}(x + 1)^2(x - 3)^2$

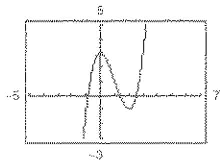
Zeros: $(-1, 0)$, $(3, 0)$

(c) and (d)



79. $f(x) = x^3 - 3x^2 + 3$

(a)



The function has three zeros. They are in the intervals $(-1, 0)$, $(1, 2)$ and $(2, 3)$.

(b) Zeros: -0.879, 1.347, 2.532

(c)

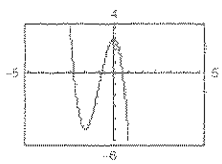
x	y_1
-0.9	-0.159
-0.89	-0.0813
-0.88	-0.0047
-0.87	0.0708
-0.86	0.14514
-0.85	0.21838
-0.84	0.2905

x	y_1
1.3	0.127
1.31	0.09979
1.32	0.07277
1.33	0.04594
1.34	0.0193
1.35	-0.0071
1.36	-0.0333

x	y_1
2.5	-0.125
2.51	-0.087
2.52	-0.0482
2.53	-0.0084
2.54	0.03226
2.55	0.07388
2.56	0.11642

80. $f(x) = -2x^3 - 6x^2 + 3$

(a)



The function has three zeros. They are in the intervals $(-3, -2)$, $(-1, 0)$, and $(0, 1)$.

(b) Zeros: -2.810, -0.832, 0.642

(c)

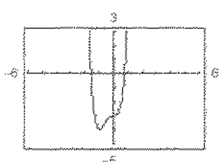
x	y_1
-2.83	0.277
-2.82	0.137
-2.81	≈ 0
-2.80	-0.136
-2.79	-0.269

x	y_1
-0.86	-0.166
-0.85	-0.107
-0.84	-0.048
-0.83	0.010
-0.82	0.068

x	y_1
0.62	0.217
0.63	0.119
0.64	0.018
0.65	-0.084
0.66	-0.189

81. $g(x) = 3x^4 + 4x^3 - 3$

(a)



The function has two zeros. They are in the intervals $(-2, -1)$ and $(0, 1)$.

(b) Zeros: -1.585, 0.779

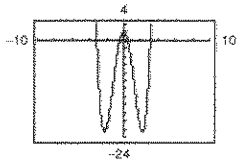
(c)

x	y_1
-1.6	0.2768
-1.59	0.09515
-1.58	-0.0812
-1.57	-0.2524
-1.56	-0.4184
-1.55	-0.5795
-1.54	-0.7356

x	y_1
0.75	-0.3633
0.76	-0.2432
0.77	-0.1193
0.78	0.00866
0.79	0.14066
0.80	0.2768
0.81	0.41717

82. $h(x) = x^4 - 10x^2 + 2$

(a)



The function has four zeros. They are in the intervals $(0, 1)$, $(3, 4)$, $(-1, 0)$ and $(-4, -3)$.

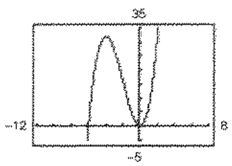
(b) Notice that h is even. Hence, the zeros come in symmetric pairs. Zeros: ± 0.452, ± 3.130

(c) Because the function is even, we only need to verify the positive zeros.

x	y_1
0.42	0.26712
0.43	0.18519
0.44	0.10148
0.45	0.01601
0.46	-0.0712
0.47	-0.1602
0.48	-0.2509

x	y_1
3.09	-2.315
3.10	-1.748
3.11	-1.171
3.12	-0.5855
3.13	0.01025
3.14	0.61571
3.15	1.231

83.

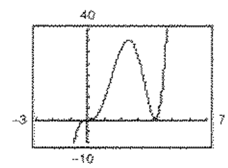


$f(x) = x^2(x + 6)$

No symmetry

Two x-intercepts

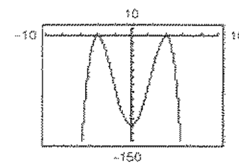
84. $h(x) = x^3(x - 4)^2$



No symmetry

Two x-intercepts $(0, 0)$, $(4, 0)$

85.

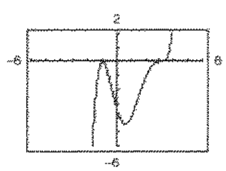


$g(t) = -\frac{1}{2}(t - 4)^2(t + 4)^2$

Symmetric about the y-axis

Two x-intercepts

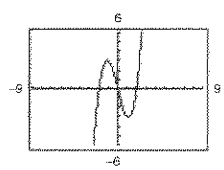
86. $g(x) = \frac{1}{8}(x + 1)^2(x - 3)^3$



No symmetry.

Two x-intercepts $(-1, 0)$, $(3, 0)$

87.



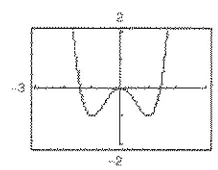
$f(x) = x^3 - 4x$

$= x(x + 2)(x - 2)$

Symmetric to origin

Three x-intercepts

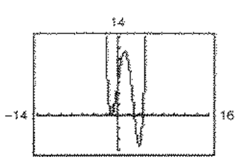
88. $f(x) = x^4 - 2x^2$



Symmetric with respect to y-axis

Three x-intercepts $(0, 0)$, $(\pm\sqrt{2}, 0)$

89.



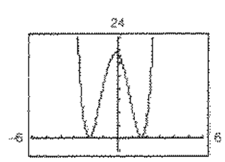
$g(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$

Three x-intercepts

No symmetry

90. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$

No symmetry; two x-intercepts



91. (a) Volume = length × width × height

Because the box is made from a square, length = width.

Thus:

Volume = $(\text{length})^2 \times \text{height} = (36 - 2x)^2 x$

(b) Domain: $0 < 36 - 2x < 36$

$-36 < -2x < 0$

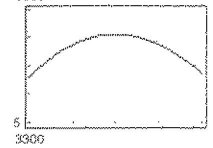
$18 > x > 0$

(c)

Height, x	Length and Width	Volume, V
1	$36 - 2(1)$	$1[36 - 2(1)]^2 = 1156$
2	$36 - 2(2)$	$2[36 - 2(2)]^2 = 2048$
3	$36 - 2(3)$	$3[36 - 2(3)]^2 = 2700$
4	$36 - 2(4)$	$4[36 - 2(4)]^2 = 3136$
5	$36 - 2(5)$	$5[36 - 2(5)]^2 = 3380$
6	$36 - 2(6)$	$6[36 - 2(6)]^2 = 3456$
7	$36 - 2(7)$	$7[36 - 2(7)]^2 = 3388$

Maximum volume 3456 for $x = 6$

(d)



$x = 6$ when $V(x)$ is maximum.

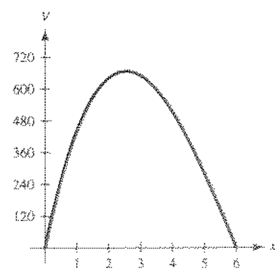
92. (a) $V(x) = \text{length} \times \text{width} \times \text{height}$

$= (24 - 2x)(24 - 4x)x$

$= 8x(12 - x)(6 - x)$

(b) Domain: $0 < x < 6$

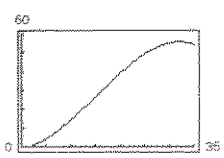
(c)



Maximum occurs at $x \approx 2.54$.

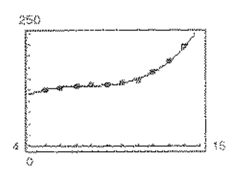
93. The point of diminishing returns (where the graph changes from curving upward to curving downward) occurs when $x = 200$. The point is (200, 160) which corresponds to spending \$2,000,000 on advertising to obtain a revenue of \$160 million.

94.



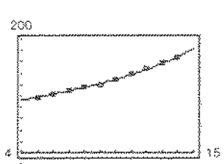
Point of Diminishing Returns: (15.2, 27.3)
15.2 years

95.



The model is a good fit.

96.



97. For 2010, $t = 20$, and

$y_1 \approx \$730.2$ thousand

$y_2 \approx \$285.0$ thousand.

Answers will vary.

98. Answers will vary.

99. True. $f(x) = x^6$ has only one zero, 0.

100. True. The degree is odd and the leading coefficient is -1.

101. False. The graph touches at $x = 1$, but does not cross the x-axis there.

102. False. The graph crosses the x-axis at $x = -3$ and $x = 0$.

103. True. The exponent of $(x + 2)$ is odd (3).

104. False. The graph rises to the left, and rises to the right.

105. The zeros are 0, 1, 1, and the graph rises to the right. Matches (b).

106. The zeros are 0, 0, 2, 2, and the graph falls to the right. Matches (e).

107. The zeros are 1, 1, -2, -2, and the graph rises to the right. Matches (a).

108. $$\begin{aligned}(f + g)(-4) &= f(-4) + g(-4)\\ &= -59 + 128 = 69\end{aligned}$$

109. $$\begin{aligned}(g - f)(3) = g(3) - f(3) &= 8(3)^2 - [14(3) - 3]\\ &= 72 - 39 = 33\end{aligned}$$

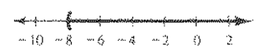
110. $$(f \cdot g)\left(-\frac{4}{7}\right) = f\left(-\frac{4}{7}\right)g\left(-\frac{4}{7}\right) = (-11)\left(\frac{8 \cdot 16}{49}\right) = -\frac{1408}{49} \approx -28.7347$$

111. $$\left(\frac{f}{g}\right)(-1.5) = \frac{f(-1.5)}{g(-1.5)} = \frac{-24}{18} = -\frac{4}{3}$$

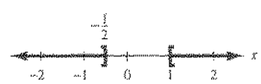
112. $(f \circ g)(-1) = f(g(-1)) = f(8) = 109$

113. $(g \circ f)(0) = g(f(0)) = g(-3) = 8(-3)^2 = 72$

114. $$\begin{aligned}3(x - 5) &< 4x - 7\\ 3x - 15 &< 4x - 7\\ -8 &< x\end{aligned}$$



115. $$\begin{aligned}2x^2 - x &\ge 1\\ 2x^2 - x - 1 &\ge 0\\ (2x + 1)(x - 1) &\ge 0\end{aligned}$$

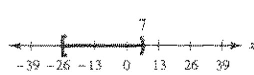


$[2x + 1 \ge 0 \text{ and } x - 1 \ge 0]$ or $[2x + 1 \le 0 \text{ and } x - 1 \le 0]$

$[x \ge -\frac{1}{2} \text{ and } x \ge 1]$ or $[x \le -\frac{1}{2} \text{ and } x \le 1]$

$x \ge 1$ or $x \le -\frac{1}{2}$

116. $$\begin{aligned}\frac{5x - 2}{x - 7} &\le 4\\ \frac{5x - 2}{x - 7} - 4 &\le 0\\ \frac{5x - 2 - 4(x - 7)}{x - 7} &\le 0\\ \frac{x + 26}{x - 7} &\le 0\end{aligned}$$

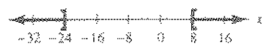


$[x + 26 \ge 0 \text{ and } x - 7 < 0]$ or $[x + 26 \le 0 \text{ and } x - 7 > 0]$

$[x \ge -26 \text{ and } x < 7]$ or $[x \le -26 \text{ and } x > 7]$

$-26 \le x < 7$ impossible

117. $|x + 8| - 1 \geq 15$



$|x + 8| \geq 16$

$x + 8 \geq 16$ or $x + 8 \leq -16$

$x \geq 8$ or $x \leq -24$

Section 2.3 Real Zeros of Polynomial Functions

You should know the following basic techniques and principles of polynomial division.

- The Division Algorithm (Long Division of Polynomials)
- Synthetic Division
- $f(k)$ is equal to the remainder of $f(x)$ divided by $(x - k)$.
- $f(k) = 0$ if and only if $(x - k)$ is a factor of $f(x)$.
- The Rational Zero Test
- The Upper and Lower Bound Rule

Vocabulary Check

1. $f(x)$is the dividend, $d(x)$ is the divisor, $q(x)$ is the quotient, and $r(x)$ is the remainder.
2. improper, proper
3. synthetic division
4. Rational Zero
5. Descartes's Rule, Signs
6. Remainder Theorem
7. upper bound, lower bound

1.

$$\begin{array}{r} 2x + 4 \\ x + 3 \,\overline{)\, 2x^2 + 10x + 12} \\ \underline{2x^2 + 6x} \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{array}$$

$$\frac{2x^2 + 10x + 12}{x + 3} = 2x + 4, x \neq -3$$

2.

$$\begin{array}{r} 5x + 3 \\ x - 4 \,\overline{)\, 5x^2 - 17x - 12} \\ \underline{5x^2 - 20x} \\ 3x - 12 \\ \underline{3x - 12} \\ 0 \end{array}$$

$$\frac{5x^2 - 17x - 12}{x - 4} = 5x + 3, x \neq 4$$

3.

$$\begin{array}{r} x^3 + 3x^2 \qquad - 1 \\ x + 2 \,\overline{)\, x^4 + 5x^3 + 6x^2 - x - 2} \\ \underline{x^4 + 2x^3} \\ 3x^3 + 6x^2 \\ \underline{3x^3 + 6x^2} \\ -x - 2 \\ \underline{-x - 2} \\ 0 \end{array}$$

$$\frac{x^4 + 5x^3 + 6x^2 - x - 2}{x + 2} = x^3 + 3x^2 - 1, x \neq -2$$

4.

$$\begin{array}{r} x^2 - x - 20 \\ x - 3 \,\overline{)\, x^3 - 4x^2 - 17x + 6} \\ \underline{x^3 - 3x^2} \\ -x^2 - 17x \\ \underline{-x^2 + 3x} \\ -20x + 6 \\ \underline{-20x + 60} \\ -54 \end{array}$$

$$\frac{x^3 - 4x^2 - 17x + 6}{x - 3} = x^2 - x - 20 - \frac{54}{x - 3}$$

5.
$$\begin{array}{r} x^2 - 3x + 1 \\ 4x + 5\overline{)\,4x^3 - 7x^2 - 11x + 5} \\ -(4x^3 + 5x^2) \\ \hline -12x^2 - 11x \\ -(-12x^2 - 15x) \\ \hline 4x + 5 \\ -(4x + 5) \\ \hline 0 \end{array}$$

$$\frac{4x^3 - 7x^2 - 11x + 5}{4x + 5} = x^2 - 3x + 1, \quad x \neq -\frac{5}{4}$$

6.
$$\begin{array}{r} x^2 \qquad - 25 \\ 2x - 3\overline{)\,2x^3 - 3x^2 - 50x + 75} \\ 2x^3 - 3x^2 \\ \hline -50x + 75 \\ -50x + 75 \\ \hline 0 \end{array}$$

$$\frac{2x^3 - 3x^2 - 50x + 75}{2x - 3} = x^2 - 25, \ x \neq \frac{3}{2}$$

7.
$$\begin{array}{r} 7x^2 - 14x + 28 \\ x + 3\overline{)\,7x^3 + 0x^2 + 0x + 3} \\ 7x^3 + 14x^2 \\ \hline -14x^2 \\ -14x^2 - 28x \\ \hline 28x + 3 \\ 28x + 56 \\ \hline -53 \end{array}$$

$$\frac{7x^3 + 3}{x + 2} = 7x^2 - 14x + 28 - \frac{53}{x + 2}$$

8.
$$\begin{array}{r} 4x^3 - 2x^2 + x - \frac{1}{2} \\ 2x + 1\overline{)\,8x^4 + 0x^3 + 0x^2 + 0x - 5} \\ 8x^4 + 4x^3 \\ \hline -4x^3 \\ -4x^3 - 2x^2 \\ \hline 2x^2 \\ 2x^2 + x \\ \hline -x - 5 \\ -x - \frac{1}{2} \\ \hline -\frac{9}{2} \end{array}$$

$$\frac{8x^4 - 5}{2x + 1} = 4x^3 - 2x^2 + x - \frac{1}{2} - \frac{9/2}{2x + 1}$$

9.
$$\begin{array}{r} 3x + 5 \\ 2x^2 + 0x + 1\overline{)\,6x^3 + 10x^2 + x + 8} \\ -(6x^3 + 0x^2 + 3x) \\ \hline 10x^2 - 2x + 8 \\ -(10x^2 + 0x + 5) \\ \hline -2x + 3 \end{array}$$

$$\frac{6x^3 + 10x^2 + x + 8}{2x^2 + 1} = 3x + 5 - \frac{2x - 3}{2x^2 + 1}$$

10.
$$\begin{array}{r} x^2 + 2x + 4 \\ x^2 - 2x + 3\overline{)\,x^4 + 0x^3 + 3x^2 + 0x + 1} \\ x^4 - 2x^3 + 3x^2 \\ \hline 2x^3 \qquad + 0x \\ 2x^3 - 4x^2 + 6x \\ \hline 4x^2 - 6x + 1 \\ 4x^2 - 8x + 12 \\ \hline 2x - 11 \end{array}$$

$$\frac{x^4 + 3x^2 + 1}{x^2 - 2x + 3} = x^2 + 2x + 4 + \frac{2x - 11}{x^2 - 2x + 3}$$

11.
$$\begin{array}{r} x \\ x^2 + 1\overline{)\,x^3 + 0x^2 + 0x - 9} \\ x^3 \qquad + x \\ \hline -x - 9 \end{array}$$

$$\frac{x^3 - 9}{x^2 + 1} = x - \frac{x + 9}{x^2 + 1}$$

12.
$$\begin{array}{r} x^2 \\ x^3 - 1\overline{)\,x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 7} \\ x^5 \qquad\qquad - x^2 \\ \hline x^2 + 7 \end{array}$$

$$\frac{x^5 + 7}{x^3 - 1} = x^2 + \frac{x^2 + 7}{x^3 - 1}$$

13.

$$\begin{array}{r} 2x \\ x^2 - 2x + 1 \overline{\smash{\big)}\, 2x^3 - 4x^2 - 15x + 5} \\ \underline{2x^3 - 4x^2 + 2x} \\ -17x + 5 \end{array}$$

$$\frac{2x^3 - 4x^2 - 15x + 5}{(x-1)^2} = 2x - \frac{17x - 5}{(x-1)^2}$$

14. $(x-1)^3 = x^3 - 3x^2 + 3x - 1$

$$\begin{array}{r} x + 3 \\ x^3 - 3x^2 + 3x - 1 \overline{\smash{\big)}\, x^4 } \\ \underline{x^4 - 3x^3 + 3x^2 - x} \\ 3x^3 - 3x^2 + x \\ \underline{3x^3 - 9x^2 + 9x - 3} \\ 6x^2 - 8x + 3 \end{array}$$

$$\frac{x^4}{(x-1)^3} = x + 3 + \frac{6x^2 - 8x + 3}{(x-1)^3}$$

15.

$$\begin{array}{r|rrrr} 5 & 3 & -17 & 15 & -25 \\ & & 15 & -10 & 25 \\ \hline & 3 & -2 & 5 & 0 \end{array}$$

$$\frac{3x^3 - 17x^2 + 15x - 25}{x - 5} = 3x^2 - 2x + 5,\ x \neq 5$$

16.

$$\begin{array}{r|rrrr} -3 & 5 & 18 & 7 & -6 \\ & & -15 & -9 & 6 \\ \hline & 5 & 3 & -2 & 0 \end{array}$$

$$\frac{5x^3 + 18x^2 + 7x - 6}{x + 3} = 5x^2 + 3x - 2,\ x \neq -3$$

17.

$$\begin{array}{r|rrrr} 3 & 6 & 7 & -1 & 26 \\ & & 18 & 75 & 222 \\ \hline & 6 & 25 & 74 & 248 \end{array}$$

$$\frac{6x^3 + 7x^2 - x + 26}{x - 3} = 6x^2 + 25x + 74 + \frac{248}{x - 3}$$

18.

$$\begin{array}{r|rrrr} -6 & 2 & 14 & -20 & 7 \\ & & -12 & -12 & 192 \\ \hline & 2 & 2 & -32 & 199 \end{array}$$

$$\frac{2x^3 + 14x^2 - 20x + 7}{x + 6} = 2x^2 + 2x - 32 + \frac{199}{x + 6}$$

19.

$$\begin{array}{r|rrrr} 2 & 9 & -18 & -16 & 32 \\ & & 18 & 0 & -32 \\ \hline & 9 & 0 & -16 & 0 \end{array}$$

$$\frac{9x^3 - 18x^2 - 16x + 32}{x - 2} = 9x^2 - 16,\ x \neq 2$$

20.

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

$$\frac{5x^3 + 6x + 8}{x + 2} = 5x^2 - 10x + 26 - \frac{44}{x + 2}$$

21.

$$\begin{array}{r|rrrr} -8 & 1 & 0 & 0 & 512 \\ & & -8 & 64 & -512 \\ \hline & 1 & -8 & 64 & 0 \end{array}$$

$$\frac{x^3 + 512}{x + 8} = x^2 - 8x + 64,\ x \neq -8$$

22.

$$\begin{array}{r|rrrr} 9 & 1 & 0 & 0 & -729 \\ & & 9 & 81 & 729 \\ \hline & 1 & 9 & 81 & 0 \end{array}$$

$$\frac{x^3 - 729}{x - 9} = x^2 + 9x + 81,\ x \neq 9$$

23.

$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 16 & -23 & -15 \\ & & -2 & -7 & 15 \\ \hline & 4 & 14 & -30 & 0 \end{array}$$

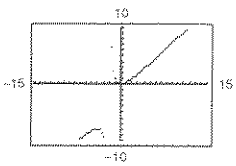
$$\frac{4x^3 + 16x^2 - 23x - 15}{x + \frac{1}{2}} = 4x^2 + 14x - 30,\ x \neq -\frac{1}{2}$$

24.

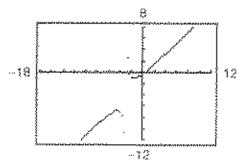
$$\begin{array}{r|rrrr} \frac{3}{2} & 3 & -4 & 0 & 5 \\ & & \frac{9}{2} & \frac{3}{4} & \frac{9}{8} \\ \hline & 3 & \frac{1}{2} & \frac{3}{4} & \frac{49}{8} \end{array}$$

$$\frac{3x^3 - 4x^2 + 5}{x - \frac{3}{2}} = 3x^2 + \frac{1}{2}x + \frac{3}{4} + \frac{49}{8x - 12}$$

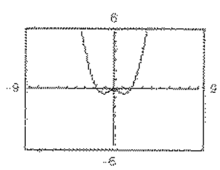
25. $$\begin{aligned} y_2 &= x - 2 + \frac{4}{x+2} \\ &= \frac{(x-2)(x+2)+4}{x+2} \\ &= \frac{x^2-4+4}{x+2} \\ &= \frac{x^2}{x+2} \\ &= y_1 \end{aligned}$$



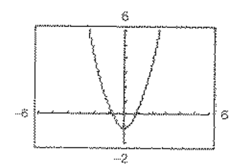
26. $$\begin{aligned} y_2 &= x - 1 + \frac{2}{x+3} \\ &= \frac{(x-1)(x+3)+2}{x+3} \\ &= \frac{x^2+2x-3+2}{x+3} \\ &= \frac{x^2+2x-1}{x+3} \\ &= y_1 \end{aligned}$$



27. $$\begin{aligned} y_2 &= x^2 - 8 + \frac{39}{x^2+5} \\ &= \frac{(x^2-8)(x^2+5)+39}{x^2+5} \\ &= \frac{x^4-8x^2+5x^2-40+39}{x^2+5} \\ &= \frac{x^4-3x^2-1}{x^2+5} \\ &= y_1 \end{aligned}$$



28. $$\begin{aligned} y_2 &= x^2 - \frac{1}{x^2+1} \\ &= \frac{x^2(x^2+1)-1}{x^2+1} \\ &= \frac{x^4+x^2-1}{x^2+1} \\ &= y_1 \end{aligned}$$



29. $f(x) = x^3 - x^2 - 14x + 11, \quad k = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$f(x) = (x-4)(x^2+3x-2)+3$

$f(4) = (0)(26) + 3 = 3$

30. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14, \quad k = -\frac{2}{3}$

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 15 & 10 & -6 & 0 & 14 \\ & & -10 & 0 & 4 & -\frac{8}{3} \\ \hline & 15 & 0 & -6 & 4 & \frac{34}{3} \end{array}$$

$f(x) = \left(x + \frac{2}{3}\right)(15x^3 - 6x + 4) + \frac{34}{3}$

$f\left(-\frac{2}{3}\right) = \frac{34}{3}$

31. $$\begin{array}{r|llll} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2+3\sqrt{2} & 6 \\ \hline & 1 & 3+\sqrt{2} & 3\sqrt{2} & -8 \end{array}$$

$f(x) = \left(x - \sqrt{2}\right)\left(x^2 + \left(3+\sqrt{2}\right)x + 3\sqrt{2}\right) - 8$

$f\left(\sqrt{2}\right) = 0\left(4 + 6\sqrt{2}\right) - 8 = -8$

32. $$\begin{array}{r|llll} -\sqrt{5} & 1 & 2 & -5 & -4 \\ & & -\sqrt{5} & 5-2\sqrt{5} & 10 \\ \hline & 1 & 2-\sqrt{5} & -2\sqrt{5} & 6 \end{array}$$

$f(x) = \left(x + \sqrt{5}\right)\left(x^2 + \left(2-\sqrt{5}\right)x - 2\sqrt{5}\right) + 6$

$f\left(-\sqrt{5}\right) = 6$

33.
$$\begin{array}{r|rrrr} 1-\sqrt{3} & 4 & -6 & -12 & -4 \\ & & 4-4\sqrt{3} & 10-2\sqrt{3} & 4 \\ \hline & 4 & -2-4\sqrt{3} & -2-2\sqrt{3} & 0 \end{array}$$

$f(x) = (x - 1 + \sqrt{3})[4x^2 - (2 + 4\sqrt{3})x - (2 + 2\sqrt{3})]$

$f(1 - \sqrt{3}) = 0$

34.
$$\begin{array}{r|rrrr} 2+\sqrt{2} & -3 & 8 & 10 & -8 \\ & & -6-3\sqrt{2} & -2-4\sqrt{2} & 8 \\ \hline & -3 & 2-3\sqrt{2} & 8-4\sqrt{2} & 0 \end{array}$$

$f(x) = (x - (2 + \sqrt{2}))(-3x^2 + (2 - 3\sqrt{2})x + 8 - 4\sqrt{2})$

$f(2 + \sqrt{2}) = 0$

35. $f(x) = 2x^3 - 7x + 3$

(a)
$$\begin{array}{r|rrrr} 1 & 2 & 0 & -7 & 3 \\ & & 2 & 2 & -5 \\ \hline & 2 & 2 & -5 & -2 \end{array} = f(1)$$

(b)
$$\begin{array}{r|rrrr} -2 & 2 & 0 & -7 & 3 \\ & & -4 & 8 & -2 \\ \hline & 2 & -4 & 1 & 1 \end{array} = f(-2)$$

(c)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 0 & -7 & 3 \\ & & 1 & \frac{1}{2} & -\frac{13}{4} \\ \hline & 2 & 1 & -\frac{13}{2} & -\frac{1}{4} \end{array} = f\left(\tfrac{1}{2}\right)$$

(d)
$$\begin{array}{r|rrrr} 2 & 2 & 0 & -7 & 3 \\ & & 4 & 8 & 2 \\ \hline & 2 & 4 & 1 & 5 \end{array} = f(2)$$

36. $g(x) = 2x^6 + 3x^4 - x^2 + 3$

(a)
$$\begin{array}{r|rrrrrrr} 2 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 4 & 8 & 22 & 44 & 86 & 172 \\ \hline & 2 & 4 & 11 & 22 & 43 & 86 & 175 \end{array} = g(2)$$

(b)
$$\begin{array}{r|rrrrrrr} 1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 2 & 2 & 5 & 5 & 4 & 4 \\ \hline & 2 & 2 & 5 & 5 & 4 & 4 & 7 \end{array} = g(1)$$

(c)
$$\begin{array}{r|rrrrrrr} 3 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 6 & 18 & 63 & 189 & 564 & 1692 \\ \hline & 2 & 6 & 21 & 63 & 188 & 564 & 1695 \end{array} = g(3)$$

(d)
$$\begin{array}{r|rrrrrrr} -1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & -2 & 2 & -5 & 5 & -4 & 4 \\ \hline & 2 & -2 & 5 & -5 & 4 & -4 & 7 \end{array} = g(-1)$$

37. $h(x) = x^3 - 5x^2 - 7x + 4$

(a)
$$\begin{array}{r|rrrr} 3 & 1 & -5 & -7 & 4 \\ & & 3 & -6 & -39 \\ \hline & 1 & -2 & -13 & -35 \end{array} = h(3)$$

(b)
$$\begin{array}{r|rrrr} 2 & 1 & -5 & -7 & 4 \\ & & 2 & -6 & -26 \\ \hline & 1 & -3 & -13 & -22 \end{array} = h(2)$$

(c)
$$\begin{array}{r|rrrr} -2 & 1 & -5 & -7 & 4 \\ & & -2 & 14 & -14 \\ \hline & 1 & -7 & 7 & -10 \end{array} = h(-2)$$

(d)
$$\begin{array}{r|rrrr} -5 & 1 & -5 & -7 & 4 \\ & & -5 & 50 & -215 \\ \hline & 1 & -10 & 43 & -211 \end{array} = h(-5)$$

38. $f(x) = 4x^4 - 16x^3 + 7x^2 + 20$

(a) $\begin{array}{r|rrrrr} 1 & 4 & -16 & 7 & 0 & 20 \\ & & 4 & -12 & -5 & -5 \\ \hline & 4 & -12 & -5 & -5 & 15 \end{array} = f(1)$

(b) $\begin{array}{r|rrrrr} -2 & 4 & -16 & 7 & 0 & 20 \\ & & -8 & 48 & -110 & 220 \\ \hline & 4 & -24 & 55 & -110 & 240 \end{array} = f(-2)$

(c) $\begin{array}{r|rrrrr} 5 & 4 & -16 & 7 & 0 & 20 \\ & & 20 & 20 & 135 & 675 \\ \hline & 4 & 4 & 27 & 135 & 695 \end{array} = f(5)$

(d) $\begin{array}{r|rrrrr} -10 & 4 & -16 & 7 & 0 & 20 \\ & & -40 & 560 & -5670 & 56{,}700 \\ \hline & 4 & -56 & 567 & -5670 & 56{,}720 \end{array} = f(-10)$

39. $\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$

$x^3 - 7x + 6 = (x - 2)(x^2 + 2x - 3)$

$= (x - 2)(x + 3)(x - 1)$

Zeros: $2, -3, 1$

40. $\begin{array}{r|rrrr} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$

$x^3 - 28x - 48 = (x + 4)(x^2 - 4x - 12)$

$= (x + 4)(x - 6)(x + 2)$

Zeros: $-4, -2, 6$

41. $\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$

$2x^3 - 15x^2 + 27x - 10$

$= \left(x - \frac{1}{2}\right)(2x^2 - 14x + 20)$

$= (2x - 1)(x - 2)(x - 5)$

Zeros: $\frac{1}{2}, 2, 5$

42. $\begin{array}{r|rrrr} \frac{2}{3} & 48 & -80 & 41 & -6 \\ & & 32 & -32 & 6 \\ \hline & 48 & -48 & 9 & 0 \end{array}$

$48x^3 - 80x^2 + 41x - 6 = \left(x - \frac{2}{3}\right)(48x^2 - 48x + 9)$

$= (3x - 2)(4x - 3)(4x - 1)$

Zeros: $\frac{2}{3}, \frac{3}{4}, \frac{1}{4}$

43. (a) $\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$

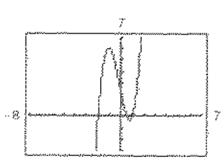
(b) $2x^2 - 3x + 1 = (2x - 1)(x - 1)$

Remaining factors: $(2x - 1)$, $(x - 1)$

(c) $f(x) = (x + 2)(2x - 1)(x - 1)$

(d) Real zeros: $-2, \frac{1}{2}, 1$

(e)



44. (a) $\begin{array}{r|rrrr} -3 & 3 & 2 & -19 & 6 \\ & & -9 & 21 & -6 \\ \hline & 3 & -7 & 2 & 0 \end{array}$

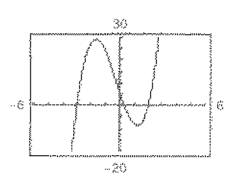
(b) $3x^2 - 7x + 2 = (3x - 1)(x - 2)$

Remaining factors: $(3x - 1)$, $(x - 2)$

(c) $f(x) = (x + 3)(3x - 1)(x - 2)$

(d) Real zeros: $-3, \frac{1}{3}, 2$

(e)



45. (a)

5	1	−4	−15	58	−40
		5	5	−50	40
	1	1	−10	8	0

−4	1	1	−10	8
		−4	12	−8
	1	−3	2	0

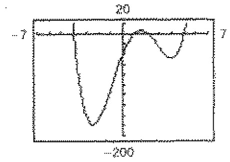
(b) $x^2 - 3x + 2 = (x - 2)(x - 1)$

Remaining factors: $(x - 2)$, $(x - 1)$

(c) $f(x) = (x - 5)(x + 4)(x - 2)(x - 1)$

(d) Real zeros: $5, -4, 2, 1$

(e)



46. (a)

−2	8	−14	−71	−10	24
		−16	60	22	−24
	8	−30	−11	12	0

4	8	−30	−11	12
		32	8	−12
	8	2	−3	0

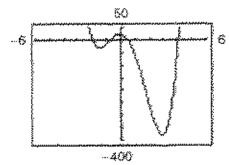
(b) $8x^2 + 2x - 3 = (4x + 3)(2x - 1)$

Remaining factors: $(4x + 3)$, $(2x - 1)$

(c) $f(x) = (x + 2)(x - 4)(4x + 3)(2x - 1)$

(d) Real zeros: $-2, 4, -\frac{3}{4}, \frac{1}{2}$

(e)



47. (a)

$-\frac{1}{2}$	6	41	−9	−14
		−3	−19	14
	6	38	−28	0

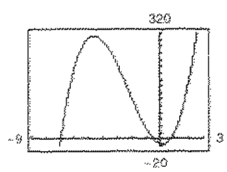
(b) $6x^2 + 38x - 28 = (3x - 2)(2x + 14)$

Remaining factors: $(3x - 2)$, $(x + 7)$

(c) $f(x) = (2x + 1)(3x - 2)(x + 7)$

(d) Real zeros: $-\frac{1}{2}, \frac{2}{3}, -7$

(e)



48. (a)

$\frac{1}{2}$	2	−1	−10	5
		1	0	−5
	2	0	−10	0

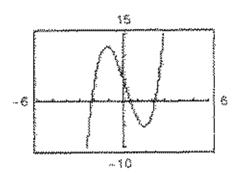
(b) $2x^2 - 10 = 2(x - \sqrt{5})(x + \sqrt{5})$

Remaining factors: $(x - \sqrt{5})$, $(x + \sqrt{5})$

(c) $f(x) = (2x - 1)(x + \sqrt{5})(x - \sqrt{5})$

(d) Real zeros: $\frac{1}{2}, \pm\sqrt{5}$

(e)



49. $f(x) = x^3 + 3x^2 - x - 3$

$p =$ factor of -3

$q =$ factor of 1

Possible rational zeros: $\pm 1, \pm 3$

$f(x) = x^2(x + 3) - (x + 3) = (x + 3)(x^2 - 1)$

Rational zeros: $\pm 1, -3$

50. $f(x) = x^3 - 4x^2 - 4x + 16$

$p =$ factor of 16

$q =$ factor of 1

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$f(x) = x^2(x - 4) - 4(x - 4) = (x - 4)(x^2 - 4)$

Rational zeros: $4, \pm 2$

51. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

$p =$ factor of -45

$q =$ factor of 2

Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45,$
$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

Using synthetic division, -1, 3, and 5 are zeros.

$f(x) = (x + 1)(x - 3)(x - 5)(2x - 3)$

Rational zeros: $-1, 3, 5, \frac{3}{2}$

52. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

$p =$ factor of -2

$q =$ factor of 4

Possible rational zeros: $\pm 2, \pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}$

Using synthetic division, -1, 1 and 2 are zeros.

$f(x) = (x + 1)(x - 1)(x - 2)(2x - 1)(2x + 1)$

Rational zeros: $\pm 1, \pm\frac{1}{2}, 2$

53. $z^4 - z^3 - 2z - 4 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \end{array} \qquad \begin{array}{r|rrrr} 2 & 1 & -2 & 2 & -4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$z^4 - z^3 - 2z - 4 = (z + 1)(z - 2)(z^2 + 2) = 0$

The only real zeros are -1 and 2. You can verify this by graphing the function $f(z) = z^4 - z^3 - 2z - 4$.

54. $x^4 - x^3 - 29x^2 - x - 30 = 0$

Using a graphing utility and synthetic division, $x = 6$ and $x = -5$ are rational zeros. Hence, $(x - 6)(x + 5)(x^2 + 1) = 0 \implies x = -5, 6$.

55. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

Using a graphing utility and synthetic division, $1/2$, 1, and -6 are rational zeros. Hence,

$(y + 6)(y - 1)^2(2y - 1) = 0 \implies y = -6, 1, \frac{1}{2}$.

56. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$

$x(x^4 - x^3 - 3x^2 + 5x - 2) = 0$

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -3 & 5 & -2 \\ & & 1 & 0 & -3 & 2 \\ \hline & 1 & 0 & -3 & 2 & 0 \end{array} \qquad \begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$x(x - 1)(x + 2)(x^2 - 2x + 1) = 0$

$x(x - 1)(x + 2)(x - 1)(x - 1) = 0$

The real zeros are $-2, 0, 1$.

57. $4x^4 - 55x^2 - 45x + 36 = 0$

Using a graphing utility and synthetic division, $4, -3, \frac{1}{2}, -\frac{3}{2}$ are rational zeros. Hence, $(x - 4)(x + 3)(2x - 1)(2x + 3) = 0 \implies$

$x = 4, -3, \frac{1}{2}, -\frac{3}{2}$.

58. $4x^4 - 43x^2 - 9x + 90 = 0$

Using a graphing utility and synthetic division, $-\frac{5}{2}$, $-2, \frac{3}{2}$, and 3 are rational zeros. Hence,

$(2x + 5)(x + 2)(2x - 3)(x - 3) = 0 \implies$

$x = -\frac{5}{2}, -2, \frac{3}{2}, 3$.

59. $4x^5 + 12x^4 - 11x^3 - 42x^2 + 7x + 30 = 0$

Using a graphing utility and synthetic division, 1, -1, -2, $\frac{3}{2}$, and $-\frac{5}{2}$ are rational zeros. Hence,
$(x - 1)(x + 1)(x + 2)(2x - 3)(2x + 5) = 0 \Longrightarrow$
$x = 1, -1, -2, \frac{3}{2}, -\frac{5}{2}$.

60. $4x^5 + 8x^4 - 15x^3 - 23x^2 + 11x + 15 = 0$

Using a graphing utility and synthetic division, 1, -1, -1, $\frac{3}{2}$, and $-\frac{5}{2}$ are rational zeros. Hence,
$(x - 1)(x + 1)^2(2x - 3)(2x + 5) = 0 \Longrightarrow$
$x = 1, -1, -1, \frac{3}{2}, -\frac{5}{2}$.

61. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) Zeros: $-2, 3.732, 0.268$

(b)
$$\begin{array}{r|rrrr} -2 & 1 & -2 & -7 & 2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & 0 \end{array} \quad t = -2 \text{ is a zero.}$$

(c) $h(t) = (t + 2)(t^2 - 4t + 1)$
$= (t + 2)\left[t - \left(\sqrt{3} + 2\right)\right]\left[t + \left(\sqrt{3} - 2\right)\right]$

62. $f(s) = s^3 - 12s^2 + 40s - 24$

(a) Zeros: $6, 5.236, 0.764$

(b)
$$\begin{array}{r|rrrr} 6 & 1 & -12 & 40 & -24 \\ & & 6 & -36 & 24 \\ \hline & 1 & -6 & 4 & 0 \end{array}$$

$f(s) = (s - 6)(s^2 - 6s + 4)$
$= (s - 6)\left(s - 3 - \sqrt{5}\right)\left(s - 3 + \sqrt{5}\right)$

63. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) $h(x) = x(x^4 - 7x^3 + 10x^2 + 14x - 24)$

From the calculator we have $x = 0, 3, 4$ and $x \approx \pm 1.414$.

(b)
$$\begin{array}{r|rrrrr} 3 & 1 & -7 & 10 & 14 & -24 \\ & & 3 & -12 & -6 & 24 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

(c) $h(x) = x(x - 3)(x - 4)(x^2 - 2)$
$= x(x - 3)(x - 4)\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)$

The exact roots are $x = 0, 3, 4, \pm\sqrt{2}$.

64. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) $x = \pm 3.0, 1.5, 0.333$

(b)
$$\begin{array}{r|rrrrr} 3 & 6 & -11 & -51 & 99 & -27 \\ & & 18 & 21 & -90 & 27 \\ \hline & 6 & 7 & -30 & 9 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 6 & 7 & -30 & 9 \\ & & -18 & 33 & -9 \\ \hline & 6 & -11 & 3 & 0 \end{array}$$

$g(x) = (x - 3)(x + 3)(6x^2 - 11x + 3)$
$= (x - 3)(x + 3)(3x - 1)(2x - 3)$

65. $f(x) = 2x^4 - x^3 + 6x^2 - x + 5$

4 variations in sign $\Longrightarrow$ 4, 2 or 0 positive real zeros

$f(-x) = 2x^4 + x^3 + 6x^2 + x + 5$

0 variations in sign $\Longrightarrow$ 0 negative real zeros

66. $f(x) = 3x^4 + 5x^3 - 6x^2 + 8x - 3$

3 sign changes $\Longrightarrow$ 3 or 1 positive zeros

$f(-x) = 3x^4 - 5x^3 - 6x^2 - 8x - 3$

1 sign change $\Longrightarrow$ 1 negative zero

67. $g(x) = 4x^3 - 5x + 8$

2 variations in sign $\Longrightarrow$ 2 or 0 positive real zeros

$g(-x) = -4x^3 + 5x + 8$

1 variation in sign $\Longrightarrow$ 1 negative real zero

68. $g(x) = 2x^3 - 4x^2 - 5$

1 sign change $\Longrightarrow$ 1 positive zero

$g(-x) = -2x^3 - 4x^2 - 5$

No sign change $\Longrightarrow$ no negative zeros

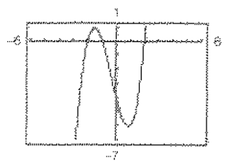
69. $f(x) = x^3 + x^2 - 4x - 4$

(a) $f(x)$ has 1 variation in sign $\implies$ 1 positive real zero.

$f(-x) = -x^3 + x^2 + 4x - 4$ has 2 variations in sign $\implies$ 2 or 0 negative real zeros.

(b) Possible rational zeros: $\pm 1, \pm 2, \pm 4$

(c)



(d) Real zeros: $-2, -1, 2$

70. (a) $f(x) = -3x^3 + 20x^2 - 36x + 16$

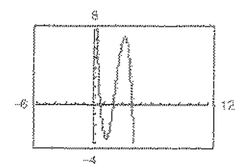
3 sign changes $\implies$ 3 or 1 positive zeros

$f(-x) = 3x^3 + 20x^2 + 36x + 16$

0 sign changes $\implies$ 0 negative zeros

(b) $\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}; \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

(c)



(d) Zeros: $\frac{2}{3}, 2, 4$

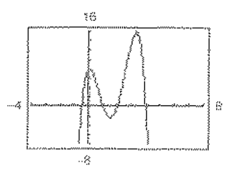
71. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) $f(x)$ has 3 variations in sign $\implies$ 3 or 1 positive real zeros.

$f(-x) = -2x^4 - 13x^3 - 21x^2 - 2x + 8$ has 1 variation in sign $\implies$ 1 negative real zero.

(b) Possible rational zeros: $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

(c)



(d) Real zeros: $-\frac{1}{2}, 1, 2, 4$

72. (a) $f(x) = 4x^4 - 17x^2 + 4$

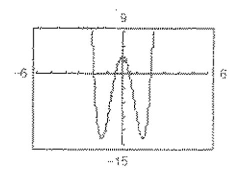
2 sign changes $\implies$ 0 or 2 positive zeros

$f(-x) = 4x^4 - 17x^2 + 4$

2 sign changes $\implies$ 0 or 2 negative zeros

(b) $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4$

(c)



(d) Zeros: $\pm 2, \pm\frac{1}{2}$

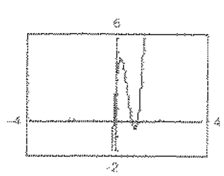
73. $f(x) = 32x^3 - 52x^2 + 17x + 3$

(a) $f(x)$ has 2 variations in sign $\implies$ 2 or 0 positive real zeros.

$f(-x) = -32x^3 - 52x^2 - 17x + 3$ has 1 variation in sign $\implies$ 1 negative real zero.

(b) Possible rational zeros: $\pm\frac{1}{32}, \pm\frac{1}{16}, \pm\frac{1}{8}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{32}, \pm\frac{3}{16}, \pm\frac{3}{8}, \pm\frac{3}{4}, \pm\frac{3}{2}, \pm 3$

(c)



(d) Real zeros: $1, \dfrac{3}{4}, -\dfrac{1}{8}$

74. (a) $f(x) = 4x^3 + 7x^2 - 11x - 18$

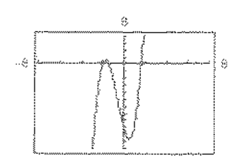
1 sign change $\implies$ 1 positive zero

$f(-x) = -4x^3 + 7x^2 + 11x - 18$

2 sign changes $\implies$ 0 or 2 negative zeros

(b) $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}$

(c)



(d) Zeros: $-2, \dfrac{1}{8} \pm \dfrac{\sqrt{145}}{8}$

75. $f(x) = x^4 - 4x^3 + 15$

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & 0 & 0 & 15 \\ & & 4 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & 15 \end{array}$$

4 is an upper bound.

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & 0 & 0 & 15 \\ & & -1 & 5 & -5 & 5 \\ \hline & 1 & -5 & 5 & -5 & 20 \end{array}$$

-1 is a lower bound.

Real zeros: 1.937, 3.705

76. $f(x) = 2x^3 - 3x^2 - 12x + 8$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -12 & 8 \\ & & 8 & 20 & 32 \\ \hline & 2 & 5 & 8 & 40 \end{array}$$

4 is an upper bound.

$$\begin{array}{r|rrrr} -3 & 2 & -3 & -12 & 8 \\ & & -6 & 27 & -45 \\ \hline & 2 & -9 & 15 & -37 \end{array}$$

-3 is a lower bound.

Real zeros: -2.152, 0.611, 3.041

77. $f(x) = x^4 - 4x^3 + 16x - 16$

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & 0 & 16 & -16 \\ & & 25 & 105 & 525 & 2705 \\ \hline & 5 & 21 & 105 & 541 & 2689 \end{array}$$

5 is an upper bound.

$$\begin{array}{r|rrrrr} -3 & 1 & -4 & 0 & 16 & -16 \\ & & -3 & 21 & -63 & 141 \\ \hline & 1 & -7 & 21 & -47 & 125 \end{array}$$

-3 is a lower bound.

Real zeros: -2, 2

78. $f(x) = 2x^4 - 8x + 3$

$$\begin{array}{r|rrrrr} 3 & 2 & 0 & 0 & -8 & 3 \\ & & 6 & 18 & 54 & 138 \\ \hline & 2 & 6 & 18 & 46 & 141 \end{array}$$

3 is an upper bound.

$$\begin{array}{r|rrrrr} -4 & 2 & 0 & 0 & -8 & 3 \\ & & -8 & 32 & -128 & 544 \\ \hline & 2 & -8 & 32 & -136 & 547 \end{array}$$

-4 is a lower bound.

Real zeros: 0.380, 1.435

79. $P(x) = x^4 - \frac{25}{4}x^2 + 9$

$= \frac{1}{4}(4x^4 - 25x^2 + 36)$

$= \frac{1}{4}(4x^2 - 9)(x^2 - 4)$

$= \frac{1}{4}(2x + 3)(2x - 3)(x + 2)(x - 2)$

The rational zeros are $\pm\frac{3}{2}$ and ± 2.

80. $f(x) = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$f(x) = \frac{1}{2}(x - 4)(2x^2 + 5x - 3)$

$= \frac{1}{2}(x - 4)(2x - 1)(x + 3)$

Rational zeros: $-3, \frac{1}{2}, 4$

81. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$

$= \frac{1}{4}(4x^3 - x^2 - 4x + 1)$

$= \frac{1}{4}[x^2(4x - 1) - 1(4x - 1)]$

$= \frac{1}{4}(4x - 1)(x^2 - 1)$

$= \frac{1}{4}(4x - 1)(x + 1)(x - 1)$

The rational zeros are $\frac{1}{4}$ and ± 1.

82. $f(z) = \frac{1}{6}(6z^3 + 11z^2 - 3z - 2)$

Possible rational zeros: $\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

$$\begin{array}{r|rrrr} -2 & 6 & 11 & -3 & -2 \\ & & -12 & 2 & 2 \\ \hline & 6 & -1 & -1 & 0 \end{array}$$

$f(x) = \frac{1}{6}(z + 2)(6z^2 - z - 1)$

$= \frac{1}{6}(z + 2)(3z + 1)(2z - 1)$

Rational zeros: $-2, -\frac{1}{3}, \frac{1}{2}$

83. $f(x) = x^3 - 1$

$= (x - 1)(x^2 + x + 1)$

Rational zeros: 1 $(x = 1)$

Irrational zeros: 0

Matches (d).

84. $f(x) = x^3 - 2$

$= \left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + \sqrt[3]{4}\right)$

Rational zeros: 0

Irrational zeros: 1, $\left(x = \sqrt[3]{2}\right)$

Matches (a).

85. $f(x) = x^3 - x = x(x + 1)(x - 1)$

Rational zeros: 3 $(x = 0, \pm 1)$

Irrational zeros: 0

Matches (b).

86. $f(x) = x^3 - 2x$

$= x(x^2 - 2)$

$= x\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$

Rational zeros: 1, $(x = 0)$

Irrational zeros: 2, $\left(x = \pm\sqrt{2}\right)$

Matches (c).

87. $y = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

Using the graph and synthetic division, $-1/2$ is a zero:

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & -9 & 5 & 3 & -1 \\ & & -1 & 5 & -5 & 1 \\ \hline & 2 & -10 & 10 & -2 & 0 \end{array}$$

$y = \left(x + \frac{1}{2}\right)(2x^3 - 10x^2 + 10x - 2)$

$x = 1$ is a zero of the cubic, so

$y = \left(2x + 1\right)(x - 1)(x^2 - 4x + 1)$.

For the quadratic term, use the Quadratic Formula.

$$x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$

The real zeros are $-\frac{1}{2}, 1, 2 \pm \sqrt{3}$.

88. $y = x^4 - 5x^3 - 7x^2 + 13x - 2$

Using the graph and synthetic division, 1 and -2 are zeros:

$y = (x - 1)(x + 2)(x^2 - 6x + 1)$

Using the Quadratic Formula:

$$x = \frac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm 2\sqrt{2}$$

The real zeros are $1, -2, 3 \pm 2\sqrt{2}$.

89. $y = -2x^4 + 17x^3 - 3x^2 - 25x - 3$

Using the graph and synthetic division, -1 and $3/2$ are zeros:

$y = -(x + 1)(2x - 3)(x^2 - 8x - 1)$

Using the Quadratic Formula:

$$x = \frac{8 \pm \sqrt{64 + 4}}{2} = 4 \pm \sqrt{17}$$

The real zeros are $-1, 3/2, 4 \pm \sqrt{17}$.

90. $y = -x^4 + 5x^3 - 10x - 4$

Using the graph and synthetic division, 2 and -1 are zeros:

$y = -(x - 2)(x + 1)(x^2 - 4x - 2)$

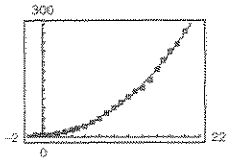
Using the Quadratic Formula:

$$x = \frac{4 \pm \sqrt{16 + 8}}{2} = 2 \pm \sqrt{6}$$

The real zeros are $2, -1, 2 \pm \sqrt{6}$.

91. (a) $P(t) = 0.0058t^3 + 0.500t^2 + 1.38t + 4.6$

(b)



(c) The model fits the data well.

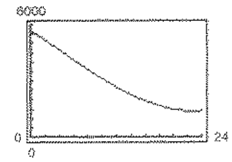
(d) For 2010, $t = 21$ and:

$$\begin{array}{r|rrrr} 21 & 0.0058 & 0.5 & 1.38 & 4.6 \\ & & 0.1218 & 13.0578 & 303.1938 \\ \hline & 0.0058 & 0.6218 & 14.4378 & 307.7938 \end{array}$$

Hence, the population will be about 307.8 million, which seems reasonable.

92. $C = 0.232t^3 - 2.11t^2 - 261.8t + 5699$

(a)



(b) For 1980, $t = 0$ and $C = 5699$ mines.

For 1990, $t = 10$ and:

$$\begin{array}{r|rrrr} 10 & 0.232 & -2.11 & -261.8 & 5699 \\ & & 2.32 & 2.1 & -2597 \\ \hline & 0.232 & 0.21 & -259.7 & 3102 \end{array}$$

3102 mines in 1990

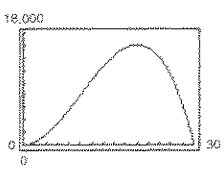
(c) Answers will vary.

93. (a) Combined length and width:

$4x + y = 120 \implies y = 120 - 4x$

$$\begin{aligned} \text{Volume} = l \cdot w \cdot h &= x^2 y \\ &= x^2(120 - 4x) \\ &= 4x^2(30 - x) \end{aligned}$$

(b)



Dimensions with maximum volume: $20 \times 20 \times 40$

(c)
$$13{,}500 = 4x^2(30 - x)$$
$$4x^3 - 120x^2 + 13{,}500 = 0$$
$$x^3 - 30x^2 + 3375 = 0$$

$$\begin{array}{r|rrrr} 15 & 1 & -30 & 0 & 3375 \\ & & 15 & -225 & -3375 \\ \hline & 1 & -15 & -225 & 0 \end{array}$$

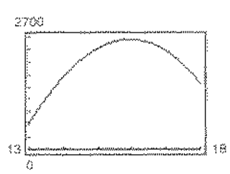
$(x - 15)(x^2 - 15x - 225) = 0$

Using the Quadratic Formula, $x = 15$ or $\dfrac{15 \pm 15\sqrt{5}}{2}$.

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

94. $y = -5.05x^3 + 3857x - 38{,}411.25,\ 13 \le x \le 18$

(a)



(b) The second air-fuel ratio of 16.89 can be obtained by finding the second point where the curves y and $y_1 = 2400$ intersect.

(c) Solve $-5.05x^3 + 3857x - 38{,}411.25 = 2400$ or $-5.05x^3 + 3857x - 40{,}811.25 = 0$.

By synthetic division:

$$\begin{array}{r|rrrr} 15 & -5.05 & 0 & 3857 & -40{,}811.25 \\ & & -75.75 & -1136.25 & 40{,}811.25 \\ \hline & -5.05 & -75.75 & 2720.75 & 0 \end{array}$$

(d) The positive zero of the quadratic $-5.05x^2 - 75.75x + 2720.75$ can be found using the Quadratic Formula.

$$x = \frac{75.75 - \sqrt{(-75.75)^2 - 4(-5.05)(2720.75)}}{2(-5.05)} \approx 16.89$$

95. False, $-\frac{4}{7}$ is a zero of f.

96.
$$\begin{array}{r|rrrrrrr} \frac{1}{2} & 6 & 1 & -92 & 45 & 184 & 4 & -48 \\ & & 3 & 2 & -45 & 0 & 92 & 48 \\ \hline & 6 & 4 & -90 & 0 & 184 & 96 & 0 \end{array}$$

True

97. The zeros are 1, 1, and -2. The graph falls to the right.

$y = a(x - 1)^2(x + 2) \quad a < 0$

Since $f(0) = -4$, $a = -2$.

$y = -2(x - 1)^2(x + 2) = -2x^3 + 6x - 4$

98. The zeros are 1, -1, and -2. The graph rises to the right.

$y = a(x - 1)(x + 1)(x + 2), a > 0$

Since $f(0) = -4$, $a = 2$.

$y = 2(x - 1)(x + 1)(x + 2) = 2x^3 + 4x^2 - 2x - 4$

99. $f(x) = -(x + 1)(x - 1)(x + 2)(x - 2)$

100. $f(x) = 2(x + 2)(x - 1)(x - 2)$

101.
$$\begin{array}{r|rrrr} 4 & 1 & -k & 2k & -8 \\ & & 4 & 16 - 4k & 64 - 8k \\ \hline & 1 & 4 - k & 16 - 2k & 56 - 8k \end{array}$$

Hence, $56 - 8k = 0 \Longrightarrow k = 7$.

102. Use synthetic division:

$$\begin{array}{r|rrrr} 3 & 1 & -k & 2k & -12 \\ & & 3 & 9 - 3k & 27 - 3k \\ \hline & 1 & 3 - k & 9 - k & 15 - 3k \end{array}$$

Since the remainder $15 - 3k$ should be 0, $k = 5$.

103. (a) $\dfrac{x^2 - 1}{x - 1} = x + 1, \quad x \neq 1$

(b) $\dfrac{x^3 - 1}{x - 1} = x^2 + x + 1, \quad x \neq 1$

(c) $\dfrac{x^4 - 1}{x - 1} = x^3 + x^2 + x + 1, \quad x \neq 1$

In general,

$$\frac{x^n - 1}{x - 1} = x^{n-1} + x^{n-2} + \cdots + x + 1, \quad x \neq 1.$$

104. You can check polynomial division by multiplying the quotient by the divisor and adding the remainder. This should yield the original dividend if the multiplication was performed correctly.

105.
$$9x^2 - 25 = 0$$
$$(3x + 5)(3x - 5) = 0$$
$$x = -\frac{5}{3}, \frac{5}{3}$$

106.
$$16x^2 - 21 = 0$$
$$x^2 = \frac{21}{16}$$
$$x = \pm\frac{\sqrt{21}}{4}$$

107. $2x^2 + 6x + 3 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{-6 \pm \sqrt{12}}{4}$$
$$= \frac{-3 \pm \sqrt{3}}{2}$$
$$x = -\frac{3}{2} + \frac{\sqrt{3}}{2}, \quad -\frac{3}{2} - \frac{\sqrt{3}}{2}$$

108.
$$8x^2 - 22x + 15 = 0$$
$$(4x - 5)(2x - 3) = 0$$
$$x = \frac{5}{4}, \frac{3}{2}$$

109. $f(x) = (x - 0)(x + 12) = x^2 + 12x$

[Answer not unique]

110. $f(x) = (x - 1)(x + 3)(x - 8)$

$= x^3 - 6x^2 - 19x + 24$

[Answer not unique]

111. $f(x) = (x - 0)(x + 1)(x - 2)(x - 5)$

$= (x^2 + x)(x^2 - 7x + 10)$

$= x^4 - 6x^3 + 3x^2 + 10x$

[Answer not unique]

112. $f(x) = \left[x - \left(2 + \sqrt{3}\right)\right]\left[x - \left(2 - \sqrt{3}\right)\right]$

$= \left[(x - 2) - \sqrt{3}\right]\left[(x - 2) + \sqrt{3}\right]$

$= (x - 2)^2 - 3$

$= x^2 - 4x + 4 - 3$

$= x^2 - 4x + 1$

[Answer not unique]

Section 2.4 Complex Numbers

- You should know how to work with complex numbers.
- Operations on complex numbers
  - (a) Addition: $(a + bi) + (c + di) = (a + c) + (b + d)i$
  - (b) Subtraction: $(a + bi) - (c + di) = (a - c) + (b - d)i$
  - (c) Multiplication: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$
  - (d) Division: $\dfrac{a + bi}{c + di} = \dfrac{a + bi}{c + di} \cdot \dfrac{c - di}{c - di} = \dfrac{ac + bd}{c^2 + d^2} + \dfrac{bc - ad}{c^2 + d^2}i$
- The complex conjugate of $a + bi$ is $a - bi$:

  $(a + bi)(a - bi) = a^2 + b^2$
- The additive inverse of $a + bi$ is $-a - bi$.
- The multiplicative inverse of $a + bi$ is

  $\dfrac{a - bi}{a^2 + b^2}$.
- $\sqrt{-a} = \sqrt{a}\,i$ for $a > 0$.

Vocabulary Check

1. (a) ii (b) iii (c) i

2. $\sqrt{-1}$, -1

3. complex, $a + bi$

4. real, imaginary

5. Mandelbrot Set

1. $a + bi = -9 + 4i$

$a = -9$

$b = 4$

2. $a + bi = 12 + 5i$

$a = 12$

$b = 5$

3. $(a - 1) + (b + 3)i = 5 + 8i$

$a - 1 = 5 \implies a = 6$

$b + 3 = 8 \implies b = 5$

4. $(a + 6) + 2bi = 6 - 5i$
$$2b = -5$$
$$b = -\tfrac{5}{2}$$
$$a + 6 = 6$$
$$a = 0$$

5. $5 + \sqrt{-16} = 5 + \sqrt{16(-1)}$
$$= 5 + 4i$$

6. $2 - \sqrt{-9} = 2 - \sqrt{9(-1)}$
$$= 2 - 3i$$

7. $-6 = -6 + 0i$

8. $8 = 8 + 0i$

9. $-5i + i^2 = -5i - 1 = -1 - 5i$

10. $-3i^2 + i = -3(-1) + i$
$$= 3 + i$$

11. $\left(\sqrt{-75}\right)^2 = -75$

12. $\left(\sqrt{-4}\right)^2 - 7 = -4 - 7$
$$= -11$$

13. $\sqrt{-0.09} = \sqrt{0.09}\,i = 0.3i$

14. $\sqrt{-0.0004} = 0.02i$

15. $(4 + i) - (7 - 2i) = (4 - 7) + (1 + 2)i$
$$= -3 + 3i$$

16. $(11 - 2i) - (-3 + 6i) = (11 + 3) + (-2 - 6)i$
$$= 14 - 8i$$

17. $\left(-1 + \sqrt{-8}\right) + \left(8 - \sqrt{-50}\right) = 7 + 2\sqrt{2}i - 5\sqrt{2}i = 7 - 3\sqrt{2}i$

18. $\left(7 + \sqrt{-18}\right) + \left(3 + \sqrt{-32}\right) = \left(7 + 3\sqrt{2}i\right) + \left(3 + 4\sqrt{2}i\right) = (7 + 3) + \left(3\sqrt{2} + 4\sqrt{2}\right)i = 10 + 7\sqrt{2}i$

19. $13i - (14 - 7i) = 13i - 14 + 7i = -14 + 20i$

20. $22 + (-5 + 8i) - 10i = (22 - 5) + (8 - 10)i = 17 - 2i$

21. $\left(\frac{3}{2} + \frac{5}{2}i\right) + \left(\frac{5}{3} + \frac{11}{3}i\right) = \left(\frac{3}{2} + \frac{5}{3}\right) + \left(\frac{5}{2} + \frac{11}{3}\right)i$
$$= \frac{9 + 10}{6} + \frac{15 + 22}{6}i$$
$$= \frac{19}{6} + \frac{37}{6}i$$

22. $\left(\frac{3}{4} + \frac{7}{5}i\right) - \left(\frac{5}{6} - \frac{1}{6}i\right) = \left(\frac{3}{4} - \frac{5}{6}\right) + \left(\frac{7}{5} + \frac{1}{6}\right)i$
$$= -\frac{1}{12} + \frac{47}{30}i$$

23. $(1.6 + 3.2i) + (-5.8 + 4.3i) = -4.2 + 7.5i$

24. $-(-3.7 - 12.8i) - \left(6.1 - \sqrt{-24.5}\right) = 3.7 + 12.8i - 6.1 + \sqrt{\frac{49}{2}}i$
$$= -2.4 + \left(12.8 + \frac{7\sqrt{2}}{2}\right)i$$
$$\approx -2.4 + 17.75i$$

25. $\sqrt{-6} \cdot \sqrt{-2} = \left(\sqrt{6}i\right)\left(\sqrt{2}i\right)$
$$= \sqrt{12}i^2 = \left(2\sqrt{3}\right)(-1) = -2\sqrt{3}$$

26. $\sqrt{-5} \cdot \sqrt{-10} = \left(\sqrt{5}i\right)\left(\sqrt{10}i\right)$
$$= \sqrt{50}i^2 = 5\sqrt{2}(-1) = -5\sqrt{2}$$

27. $\left(\sqrt{-10}\right)^2 = \left(\sqrt{10}i\right)^2 = 10i^2 = -10$

28. $\left(\sqrt{-75}\right)^2 = \left(\sqrt{75}i\right)^2 = 75i^2 = -75$

29. $(1 + i)(3 - 2i) = 3 - 2i + 3i - 2i^2$
$= 3 + i + 2$
$= 5 + i$

30. $(6 - 2i)(2 - 3i) = 12 - 18i - 4i + 6i^2$
$= 12 - 22i - 6$
$= 6 - 22i$

31. $4i(8 + 5i) = 32i + 20i^2$
$= 32i + 20(-1)$
$= -20 + 32i$

32. $-3i(6 - i) = -18i - 3 = -3 - 18i$

33. $(\sqrt{14} + \sqrt{10}\,i)(\sqrt{14} - \sqrt{10}\,i) = 14 - 10i^2 = 14 + 10 = 24$

34. $(3 + \sqrt{-5})(7 - \sqrt{-10}) = (3 + \sqrt{5}i)(7 - \sqrt{10}i)$
$= 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2$
$= 21 + \sqrt{50} + 7\sqrt{5}i - 3\sqrt{10}i$
$= (21 + 5\sqrt{2}) + (7\sqrt{5} - 3\sqrt{10})i$

35. $(4 + 5i)^2 - (4 - 5i)^2 = [(4 + 5i) + (4 - 5i)][(4 + 5i) - (4 - 5i)] = 8(10i) = 80i$

36. $(1 - 2i)^2 - (1 + 2i)^2 = 1 - 4i + 4i^2 - (1 + 4i + 4i^2)$
$= 1 - 4i + 4i^2 - 1 - 4i - 4i^2$
$= -8i$

37. $4 - 3i$ is the complex conjugate of $4 + 3i$.
$(4 + 3i)(4 - 3i) = 16 + 9 = 25$

38. The conjugate of $7 - 5i$ is $7 + 5i$.
$(7 - 5i)(7 + 5i) = 49 + 25 = 74$

39. $-6 + \sqrt{5}i$ is the complex conjugate of $-6 - \sqrt{5}i$.
$(-6 - \sqrt{5}i)(-6 + \sqrt{5}i) = 36 + 5 = 41$

40. The conjugate of $-3 + \sqrt{2}i$ is $-3 - \sqrt{2}i$.
$(-3 + \sqrt{2}i)(-3 - \sqrt{2}i) = 9 + 2 = 11$

41. $-\sqrt{20}i$ is the complex conjugate of $\sqrt{-20} = \sqrt{20}i$.
$(\sqrt{20}i)(-\sqrt{20}i) = 20$

42. The conjugate of $\sqrt{-13} = \sqrt{13}i$ is $-\sqrt{13}i$.
$\sqrt{-13}\,(-\sqrt{13}i) = (\sqrt{13}i)(-\sqrt{13}i) = 13$

43. $3 + \sqrt{2}i$ is the complex conjugate of $3 - \sqrt{-2} = 3 - \sqrt{2}i$.
$(3 - \sqrt{2}i)(3 + \sqrt{2}i) = 9 + 2 = 11$

44. The conjugate of $1 + \sqrt{-8} = 1 + 2\sqrt{2}i$ is $1 - 2\sqrt{2}i$.
$(1 + 2\sqrt{2}i)(1 - 2\sqrt{2}i) = 1 + 8 = 9$

45. $\dfrac{6}{i} = \dfrac{6}{i} \cdot \dfrac{-i}{-i} = \dfrac{-6i}{-i^2} = \dfrac{-6i}{1} = -6i$

46. $\dfrac{-5}{2i} \cdot \dfrac{i}{i} = \dfrac{-5i}{-2} = \dfrac{5}{2}i$

47. $\dfrac{2}{4 - 5i} = \dfrac{2}{4 - 5i} \cdot \dfrac{4 + 5i}{4 + 5i} = \dfrac{8 + 10i}{16 + 25} = \dfrac{8}{41} + \dfrac{10}{41}i$

48. $\dfrac{3}{1 - i} \cdot \dfrac{1 + i}{1 + i} = \dfrac{3 + 3i}{1 - i^2} = \dfrac{3 + 3i}{2} = \dfrac{3}{2} + \dfrac{3}{2}i$

49. $\dfrac{2+i}{2-i} = \dfrac{2+i}{2-i}\cdot\dfrac{2+i}{2+i}$
$= \dfrac{4+4i+i^2}{4+1}$
$= \dfrac{3+4i}{5} = \dfrac{3}{5}+\dfrac{4}{5}i$

50. $\dfrac{8-7i}{1-2i}\cdot\dfrac{1+2i}{1+2i} = \dfrac{8+16i-7i-14i^2}{1-4i^2}$
$= \dfrac{22+9i}{5} = \dfrac{22}{5}+\dfrac{9}{5}i$

51. $\dfrac{i}{(4-5i)^2} = \dfrac{i}{16-25-40i}$
$= \dfrac{i}{-9-40i}\cdot\dfrac{-9+40i}{-9+40i}$
$= \dfrac{-40-9i}{81+40^2}$
$= \dfrac{-40}{1681}-\dfrac{9}{1681}i$

52. $\dfrac{5i}{(2+3i)^2} = \dfrac{5i}{-5+12i}\cdot\dfrac{-5-12i}{-5-12i}$
$= \dfrac{-25i+60}{25+144}$
$= \dfrac{60}{169}-\dfrac{25}{169}i$

53. $\dfrac{2}{1+i}-\dfrac{3}{1-i} = \dfrac{2(1-i)-3(1+i)}{(1+i)(1-i)}$
$= \dfrac{2-2i-3-3i}{1+1}$
$= \dfrac{-1-5i}{2}$
$= -\dfrac{1}{2}-\dfrac{5}{2}i$

54. $\dfrac{2i}{2+i}+\dfrac{5}{2-i} = \dfrac{2i(2-i)}{(2+i)(2-i)}+\dfrac{5(2+i)}{(2+i)(2-i)}$
$= \dfrac{4i-2i^2+10+5i}{4-i^2}$
$= \dfrac{12+9i}{5}$
$= \dfrac{12}{5}+\dfrac{9}{5}i$

55. $\dfrac{i}{3-2i}+\dfrac{2i}{3+8i} = \dfrac{3i+8i^2+6i-4i^2}{(3-2i)(3+8i)}$
$= \dfrac{-4+9i}{9+18i+16}$
$= \dfrac{-4+9i}{25+18i}\cdot\dfrac{25-18i}{25-18i}$
$= \dfrac{-100+72i+225i+162}{25^2+18^2}$
$= \dfrac{62+297i}{949}$
$= \dfrac{62}{949}+\dfrac{297}{949}i$

56. $\dfrac{1+i}{i}-\dfrac{3}{4-i} = \dfrac{1+i}{i}\cdot\dfrac{-i}{-i}-\dfrac{3}{4-i}\cdot\dfrac{4+i}{4+i}$
$= \dfrac{-i+1}{1}-\dfrac{12+3i}{16+1}$
$= \dfrac{5}{17}-\dfrac{20}{17}i$

57. $-6i^3+i^2 = -6i^2i+i^2 = -6(-1)i+(-1) = 6i-1 = -1+6i$

58. $4i^2-2i^3 = -4+2i$

59. $\left(\sqrt{-75}\right)^3 = \left(5\sqrt{3}i\right)^3 = 5^3\left(\sqrt{3}\right)^3 i^3 = 125\left(3\sqrt{3}\right)(-i)$
$= -375\sqrt{3}i$

60. $\left(\sqrt{-2}\right)^6 = \left(\sqrt{2}i\right)^6 = 8i^6 = 8i^4i^2 = -8$

61. $\dfrac{1}{i^3} = \dfrac{1}{i^3}\cdot\dfrac{i}{i} = \dfrac{i}{i^4} = \dfrac{i}{1} = i$

62. $\dfrac{1}{(2i)^3} = \dfrac{1}{8i^3} = \dfrac{1}{-8i}\cdot\dfrac{8i}{8i} = \dfrac{8i}{-64i^2} = \dfrac{1}{8}i$

63. $(2)^3 = 8$

$$\begin{aligned}\left(-1+\sqrt{3}i\right)^3 &= (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3\\ &= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3\\ &= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i\\ &= 8\end{aligned}$$

$$\begin{aligned}\left(-1-\sqrt{3}i\right)^3 &= (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3\\ &= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3\\ &= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i\\ &= 8\end{aligned}$$

The three numbers are cube roots of 8.

64. (a) $2^4 = 16$ (b) $(-2)^4 = 16$

(c) $(2i)^4 = 2^4 i^4 = 16(1) = 16$ (d) $(-2i)^4 = (-2)^4 i^4 = 16(1) = 16$

65. $4 + 3i$

66. $-1 - 2i$

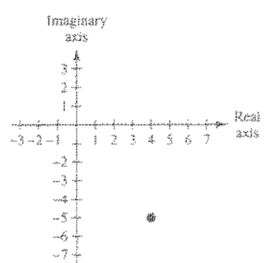
67. $5i$

68. -3

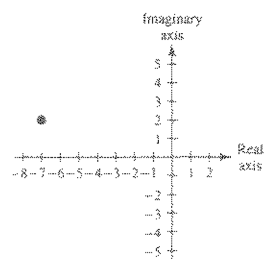
69. 2

70. $-4i$

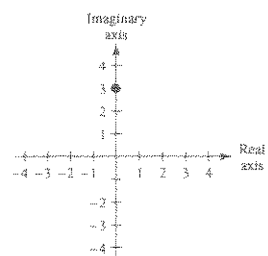
71. $4 - 5i$



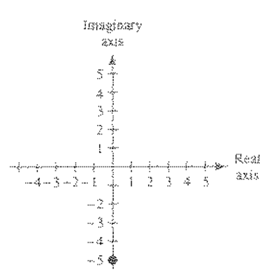
72. $-7 + 2i$



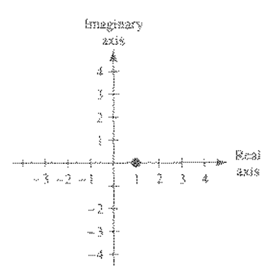
73. $3i$



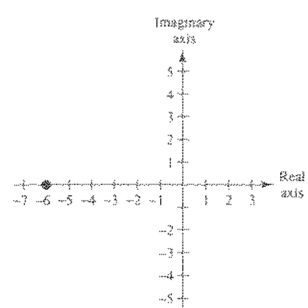
74. $-5i$



75. 1



76. -6



77. The complex number $\frac{1}{2}i$, is in the Mandelbrot Set since for $c = \frac{1}{2}i$, the corresponding Mandelbrot sequence is

$$\frac{1}{2}i,\ -\frac{1}{4} + \frac{1}{2}i,\ -\frac{3}{16} + \frac{1}{4}i,\ -\frac{7}{256} + \frac{13}{32}i,\ -\frac{10{,}767}{65{,}536} + \frac{1957}{4096}i,\ -\frac{864{,}513{,}055}{4{,}294{,}967{,}296} + \frac{46{,}037{,}845}{134{,}217{,}728}i$$

which is bounded. Or in decimal form

$0.5i, -0.25 + 0.5i, -0.1875 + 0.25i, -0.02734 + 0.40625i, -0.164291 + 0.477783i, -0.201285 + 0.343009i.$

78. 2

$2^2 + 2 = 6$

$6^2 + 2 = 38$

$38^2 + 2 = 1446$

$1446^2 + 2 = 2{,}090{,}918$

4.4×10^{12}

Not bounded. $c = 2$ is not in the Mandelbrot Set.

79. $z_1 = 5 + 2i$

$z_2 = 3 - 4i$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{5 + 2i} + \frac{1}{3 - 4i}$$

$$= \frac{(3 - 4i) + (5 + 2i)}{(5 + 2i)(3 - 4i)}$$

$$= \frac{8 - 2i}{23 - 14i}$$

$$z = \frac{23 - 14i}{8 - 2i}\left(\frac{8 + 2i}{8 + 2i}\right)$$

$$= \frac{212 - 66i}{68} \approx 3.118 - 0.971i$$

80. $z_1 = 16i + 9$

$z_2 = 20 - 10i$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{9 + 16i} + \frac{1}{20 - 10i}$$

$$= \frac{(20 - 10i) + (9 + 16i)}{(9 + 16i)(20 - 10i)}$$

$$= \frac{29 + 6i}{340 + 230i}$$

$$z = \frac{340 + 230i}{29 + 6i}\left(\frac{29 - 6i}{29 - 6i}\right) = \frac{11240 + 4630i}{877} \approx 12.816 + 5.279i$$

81. False. A real number $a + 0i = a$ is equal to its conjugate.

82. False. $i^{44} + i^{150} - i^{74} - i^{109} + i^{61} = 1 - 1 + 1 - i + i = 1$

83. False. For example, $(1 + 2i) + (1 - 2i) = 2$, which is not an imaginary number.

84. False. For example, $(i)(i) = -1$, which is not an imaginary number.

85. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$. Then

$$\overline{z_1 z_2} = \overline{(a_1 + b_1 i)(a_2 + b_2 i)}$$

$$= \overline{(a_1 a_2 - b_1 b_2) + (a_1 b_2 + b_1 a_2)i}$$

$$= (a_1 a_2 - b_1 b_2) - (a_1 b_2 + b_1 a_2)i$$

$$= (a_1 - b_1 i)(a_2 - b_2 i)$$

$$= \overline{a_1 + b_1 i}\ \overline{a_2 + b_2 i}$$

$$= \overline{z_1}\ \overline{z_2}.$$

86. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$. Then

$$\overline{z_1 + z_2} = \overline{(a_1 + b_1 i) + (a_2 + b_2 i)}$$

$$= \overline{(a_1 + a_2) + (b_1 + b_2)i}$$

$$= (a_1 + a_2) - (b_1 + b_2)i$$

$$= (a_1 - b_1 i) + (a_2 - b_2 i)$$

$$= \overline{a_1 + b_1 i} + \overline{a_2 + b_2 i}$$

$$= \overline{z_1} + \overline{z_2}.$$

87. $(4x - 5)(4x + 5) = 16x^2 - 20x + 20x - 25$
$= 16x^2 - 25$

88. $(x + 2)^3 = x^3 + 3x^2 2 + 3x(2)^2 + 2^3$
$= x^3 + 6x^2 + 12x + 8$

89. $\left(3x - \frac{1}{2}\right)(x + 4) = 3x^2 - \frac{1}{2}x + 12x - 2$
$= 3x^2 + \frac{23}{2}x - 2$

90. $(2x - 5)^2 = 4x^2 - 20x + 25$

Section 2.5 The Fundamental Theorem of Algebra

- You should know that if f is a polynomial of degree $n > 0$, then f has at least one zero in the complex number system. (Fundamental Theorem of Algebra)
- You should know that if $a + bi$ is a complex zero of a polynomial f, with real coefficients, then $a - bi$ is also a complex zero of f.
- You should know the difference between a factor that is irreducible over the rationals (such as $x^2 - 7$) and a factor that is irreducible over the reals (such as $x^2 + 9$).

Vocabulary Check

1. Fundamental Theorem, Algebra
2. Linear Factorization Theorem
3. irreducible, reals
4. complex conjugate

1. $f(x) = x^2(x + 3)$

The three zeros are $x = 0$, $x = 0$ and $x = -3$.

2. $g(x) = (x - 2)(x + 4)^3$

Zeros: $2, -4, -4, -4$

3. $f(x) = (x + 9)(x + 4i)(x - 4i)$

Zeros: $-9, \pm 4i$

4. $h(t) = (t - 3)(t - 2)(t - 3i)(t + 3i)$

The four zeros are $t = 3, 2, 3i, -3i$.

5. $f(x) = x^3 - 4x^2 + x - 4 = x^2(x - 4) + 1(x - 4) = (x - 4)(x^2 + 1)$

Zeros: $4, \pm i$

The only real zero of $f(x)$ is $x = 4$. This corresponds to the x-intercept of $(4, 0)$ on the graph.

6. $f(x) = x^3 - 4x^2 - 4x + 16$
$= x^2(x - 4) - 4(x - 4)$
$= (x^2 - 4)(x - 4)$
$= (x + 2)(x - 2)(x - 4)$

The zeros are $x = 2, -2$, and 4. This corresponds to the x-intercepts of $(-2, 0)$, $(2, 0)$, and $(4, 0)$ on the graph.

7. $f(x) = x^4 + 4x^2 + 4 = (x^2 + 2)^2$

Zeros: $\pm\sqrt{2}i, \pm\sqrt{2}i$

$f(x)$ has no real zeros and the graph of $f(x)$ has no x-intercepts.

8. $f(x) = x^4 - 3x^2 - 4$

$= (x^2 - 4)(x^2 + 1)$

$= (x + 2)(x - 2)(x^2 + 1)$

Zeros: $\pm 2, \pm i$

The only real zeros are $x = -2, 2$. This corresponds to the x-intercepts of $(-2, 0)$ and $(2, 0)$ on the graph.

9. $h(x) = x^2 - 4x + 1$

h has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}.$$

$$h(x) = [x - (2 + \sqrt{3})][x - (2 - \sqrt{3})]$$
$$= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$$

10. $g(x) = x^2 + 10x + 23$

Zeros: $x = \dfrac{-10 \pm \sqrt{8}}{2} = -5 \pm \sqrt{2}$

$g(x) = (x + 5 + \sqrt{2})(x + 5 - \sqrt{2})$

11. $f(x) = x^2 - 12x + 26$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(26)}}{2} = 6 \pm \sqrt{10}.$$

$$f(x) = [x - (6 + \sqrt{10})][x - (6 - \sqrt{10})]$$
$$= (x - 6 - \sqrt{10})(x - 6 + \sqrt{10})$$

12. $f(x) = x^2 + 6x - 2$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-2)}}{2} = -3 \pm \sqrt{11}.$$

$$f(x) = (x - (-3 + \sqrt{11}))(x - (-3 - \sqrt{11}))$$
$$= (x + 3 - \sqrt{11})(x + 3 + \sqrt{11})$$

13. $f(x) = x^2 + 25$

$= (x + 5i)(x - 5i)$

The zeros of $f(x)$ are $x = \pm 5i$.

14. $f(x) = x^2 + 36$

Zeros: $\pm 6i$

$f(x) = (x + 6i)(x - 6i)$

15. $f(x) = 16x^4 - 81$

$= (4x^2 - 9)(4x^2 + 9)$

$= (2x - 3)(2x + 3)(2x + 3i)(2x - 3i)$

Zeros: $\pm\frac{3}{2}, \pm\frac{3}{2}i$

16. $f(y) = 81y^4 - 625$

$= (9y^2 + 25)(9y^2 - 25)$

$= (3y + 5i)(3y - 5i)(3y + 5)(3y - 5)$

Zeros: $\pm\frac{5}{3}, \pm\frac{5}{3}i$

17. $f(z) = z^2 - z + 56$

$$z = \frac{1 \pm \sqrt{1 - 4(56)}}{2}$$
$$= \frac{1 \pm \sqrt{-223}}{2}$$
$$= \frac{1}{2} \pm \frac{\sqrt{223}}{2}i$$

$$f(z) = \left(z - \frac{1}{2} + \frac{\sqrt{223}i}{2}\right)\left(z - \frac{1}{2} - \frac{\sqrt{223}i}{2}\right)$$

18. $h(x) = x^2 - 4x - 3$

$$x = \frac{4 \pm \sqrt{16 + 12}}{2} = 2 \pm \sqrt{7}$$

Zeros: $2 \pm \sqrt{7}$

$h(x) = (x - 2 + \sqrt{7})(x - 2 - \sqrt{7})$

19. $f(x) = x^4 + 10x^2 + 9$

$= (x^2 + 1)(x^2 + 9)$

$= (x + i)(x - i)(x + 3i)(x - 3i)$

The zeros of $f(x)$ are $x = \pm i$ and $x = \pm 3i$.

20. $f(x) = x^4 + 29x^2 + 100$

$= (x^2 + 25)(x^2 + 4)$

Zeros: $x = \pm 2i, \pm 5i$

$f(x) = (x + 2i)(x - 2i)(x + 5i)(x - 5i)$

21. $f(x) = 3x^3 - 5x^2 + 48x - 80$

Using synthetic division, $\frac{5}{3}$ is a zero:

$$\begin{array}{r|rrrr} \frac{5}{3} & 3 & -5 & 48 & -80 \\ & & 5 & 0 & 80 \\ \hline & 3 & 0 & 48 & 0 \end{array}$$

$f(x) = \left(x - \frac{5}{3}\right)(3x^2 + 48)$

$= (3x - 5)(x^2 + 16)$

$= (3x - 5)(x + 4i)(x - 4i)$

The zeros are $\frac{5}{3}$, $4i$, $-4i$.

22. $f(x) = 3x^3 - 2x^2 + 75x - 50$

Using synthetic division, $\frac{2}{3}$ is a zero:

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & 75 & -50 \\ & & 2 & 0 & 50 \\ \hline & 3 & 0 & 75 & 0 \end{array}$$

$f(x) = \left(x - \frac{2}{3}\right)(3x^3 + 75)$

$= (3x - 2)(x^2 + 25)$

$= (3x - 2)(x + 5i)(x - 5i)$

The zeros are $\frac{2}{3}$, $5i$, $-5i$.

23. $f(t) = t^3 - 3t^2 - 15t + 125$

Possible rational zeros: $\pm 1, \pm 5, \pm 25, \pm 125$

$$\begin{array}{r|rrrr} -5 & 1 & -3 & -15 & 125 \\ & & -5 & 40 & -125 \\ \hline & 1 & -8 & 25 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $t^2 - 8t + 25$ are

$$t = \frac{8 \pm \sqrt{64 - 100}}{2} = 4 \pm 3i.$$

The zeros of $f(t)$ are $t = -5$ and $t = 4 \pm 3i$.

$f(t) = [t - (-5)][t - (4 + 3i)][t - (4 - 3i)]$

$= (t + 5)(t - 4 - 3i)(t - 4 + 3i)$

24. $f(x) = x^3 + 11x^2 + 39x + 29$

$$\begin{array}{r|rrrr} -1 & 1 & 11 & 39 & 29 \\ & & -1 & -10 & -29 \\ \hline & 1 & 10 & 29 & 0 \end{array}$$

Zeros: $x = -1, \dfrac{-10 \pm \sqrt{16}i}{2} = -5 \pm 2i$

$f(x) = (x + 1)(x + 5 + 2i)(x + 5 - 2i)$

25. $f(x) = 5x^3 - 9x^2 + 28x + 6$

Possible rational zeros: $\pm 6, \pm \frac{6}{5}, \pm 3, \pm \frac{3}{5}, \pm 2, \pm \frac{2}{5}, \pm 1, \pm \frac{1}{5}$

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 28 & 6 \\ & & -1 & 2 & -6 \\ \hline & 5 & -10 & 30 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $5x^2 - 10x + 30$ are those of $x^2 - 2x + 6$:

$$x = \frac{2 \pm \sqrt{4 - 4(6)}}{2} = 1 \pm \sqrt{5}i$$

Zeros: $-\frac{1}{5}, 1 \pm \sqrt{5}i$

$$f(x) = 5\left(x + \frac{1}{5}\right)\left(x - \left(1 + \sqrt{5}i\right)\right)\left(x - \left(1 - \sqrt{5}i\right)\right) = (5x + 1)\left(x - 1 - \sqrt{5}i\right)\left(x - 1 + \sqrt{5}i\right)$$

26. $f(s) = 3s^3 - 4s^2 + 8s + 8$

$= (3s + 2)(s^2 - 2s + 4)$

Factoring the quadratic,

$$s = \frac{2 \pm \sqrt{4 - 16}}{2} = 1 \pm \sqrt{3}i.$$

Zeros: $-\frac{2}{3}, 1 \pm \sqrt{3}i$

$f(s) = (3s + 2)(s - 1 + \sqrt{3}i)(s - 1 - \sqrt{3}i)$

27. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline 2 & 1 & -2 & 4 & -8 & 0 \\ & & 2 & 0 & 8 & \\ \hline & 1 & 0 & 4 & 0 & \end{array}$$

$g(x) = (x - 2)(x - 2)(x^2 + 4)$

$= (x - 2)^2(x + 2i)(x - 2i)$

The zeros of g are 2, 2, and $\pm 2i$.

28. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & & -3 & -9 & -3 & -9 \\ \hline -3 & 1 & 3 & 1 & 3 & 0 \\ & & -3 & 0 & -3 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

Zeros: $x = -3, \pm i$

$h(x) = (x + 3)^2(x + i)(x - i)$

29. (a) $f(x) = x^2 - 14x + 46.$

By the Quadratic Formula,

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(46)}}{2} = 7 \pm \sqrt{3}.$$

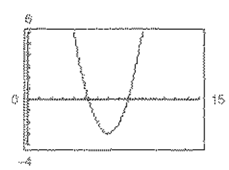
The zeros are $7 + \sqrt{3}$ and $7 - \sqrt{3}$.

(b) $f(x) = [x - (7 + \sqrt{3})][x - (7 - \sqrt{3})]$

$= (x - 7 - \sqrt{3})(x - 7 + \sqrt{3})$

(c) x-intercepts: $(7 + \sqrt{3}, 0)$ and $(7 - \sqrt{3}, 0)$

(d)



30. (a) $f(x) = x^2 - 12x + 34$

By the Quadratic Formula,

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(34)}}{2} = 6 \pm \sqrt{2}.$$

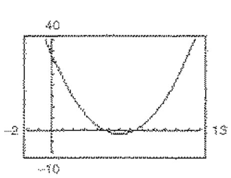
The zeros are $6 + \sqrt{2}$ and $6 - \sqrt{2}$.

(b) $f(x) = (x - (6 + \sqrt{2}))(x - (6 - \sqrt{2}))$

$= (x - 6 - \sqrt{2})(x - 6 + \sqrt{2})$

(c) x-intercepts: $(6 + \sqrt{2}, 0)(6 - \sqrt{2}, 0)$

(d)



31. (a) $f(x) = 2x^3 - 3x^2 + 8x - 12$

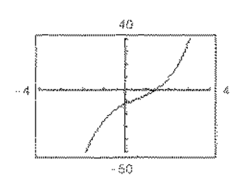
$= (2x - 3)(x^2 + 4)$

The zeros are $\frac{3}{2}, \pm 2i$.

(b) $f(x) = (2x - 3)(x + 2i)(x - 2i)$

(c) x-intercept: $\left(\frac{3}{2}, 0\right)$

(d)



32. (a) $f(x) = 2x^3 - 5x^2 + 18x - 45$

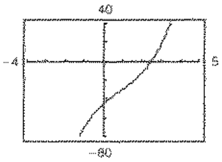
$= (2x - 5)(x^2 + 9)$

The zeros are $\frac{5}{2}, \pm 3i$.

(b) $f(x) = (2x - 5)(x + 3i)(x - 3i)$

(c) x-intercept: $\left(\frac{5}{2}, 0\right)$

(d)



33. (a) $f(x) = x^3 - 11x + 150$

$= (x + 6)(x^2 - 6x + 25)$

Use the Quadratic Formula to find the zeros of $x^2 - 6x + 25$.

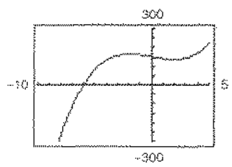
$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(25)}}{2} = 3 \pm 4i.$$

The zeros are -6, $3 + 4i$, and $3 - 4i$.

(b) $f(x) = (x + 6)(x - 3 + 4i)(x - 3 - 4i)$

(c) x-intercept: $(-6, 0)$

(d)



34. (a) $f(x) = x^3 + 10x^2 + 33x + 34$

$= (x + 2)(x^2 + 8x + 17)$

Use the Quadratic Formula to find the zeros of $x^2 + 8x + 17$.

$$x = \frac{-8 \pm \sqrt{8^2 - 4(17)}}{2}$$

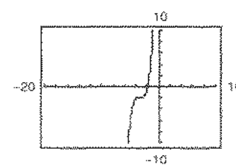
$$= \frac{-8 \pm \sqrt{-4}}{2} = -4 + i$$

The zeros are -2, $-4 + i$, and $-4 - i$.

(b) $f(x) = (x + 2)(x + 4 + i)(x + 4 - i)$

(c) x-intercept: $(-2, 0)$

(d)



35. (a) $f(x) = x^4 + 25x^2 + 144$

$= (x^2 + 9)(x^2 + 16)$

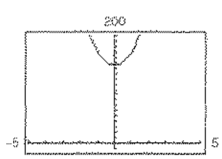
The zeros are $\pm 3i, \pm 4i$.

(b) $f(x) = (x^2 + 9)(x^2 + 16)$

$= (x + 3i)(x - 3i)(x + 4i)(x - 4i)$

(c) No x-intercepts

(d)



36. (a) $f(x) = x^4 - 8x^3 + 17x^2 - 8x + 16$

$= (x^2 + 1)(x^2 - 8x + 16)$

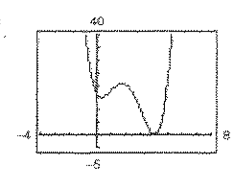
$= (x^2 + 1)(x - 4)^2$

The zeros are i, $-i$, 4 and 4.

(b) $f(x) = (x^2 + 1)(x - 4)^2$

(c) x-intercept: $(4, 0)$

(d)



37. $f(x) = (x - 2)(x - i)(x + i)$

$= (x - 2)(x^2 + 1)$

$= (x^3 - 2x^2 + x - 2)$

Note that $f(x) = a(x^3 - 2x^2 + x - 2)$, where a is any nonzero real number, has zeros $2, \pm i$.

38. $f(x) = (x - 3)(x - 4i)(x + 4i)$

$= (x - 3)(x^2 + 16)$

$= x^3 - 3x^2 + 16x - 48$

Note that $f(x) = a(x^3 - 3x^2 + 16x - 48)$, where a is any nonzero real number, has zeros $3, \pm 4i$.

39. $f(x) = (x-2)^2(x-4-i)(x-4+i)$

$= (x-2)^2(x-8x+16+1)$

$= (x^2-4x+4)(x^2-8x+17)$

$= x^4-12x^3+53x^2-100x+68$

Note that $f(x) = a(x^4-12x^3+53x^2-100x+68)$, where a is any nonzero real number, has zeros 2, 2, $4 \pm i$.

40. Because $2+5i$ is a zero, so is $2-5i$.

$f(x) = (x+1)^2(x-2-5i)(x-2+5i)$

$= (x+1)^2(x^2-4x+4+25)$

$= (x^2+2x+1)(x^2-4x+29)$

$= x^4-2x^3+22x^2+54x+29$

Note that $f(x) = a(x^4-2x^3+22x^2+54x+29)$, where a is any nonzero real number, has zeros -1, -1, $2 \pm 5i$.

41. Because $1+\sqrt{2}i$ is a zero, so is $1-\sqrt{2}i$.

$f(x) = (x-0)(x+5)\left(x-1-\sqrt{2}i\right)\left(x-1+\sqrt{2}i\right)$

$= (x^2+5x)(x^2-2x+1+2)$

$= (x^2+5x)(x^2-2x+3)$

$= x^4+3x^3-7x^2+15x$

Note that $f(x) = a(x^4+3x^3-7x^2+15x)$, where a is any nonzero real number, has zeros $0, -5, 1 \pm \sqrt{2}i$.

42. Because $1+\sqrt{2}i$ is a zero, so is $1 \pm \sqrt{2}i$.

$f(x) = (x-0)(x-4)\left(x-1-\sqrt{2}i\right)\left(x-1+\sqrt{2}i\right)$

$= (x^2-4x)(x^2-2x+1+2)$

$= x^4-6x^3+11x^2-12x$

Note that $f(x) = a(x^4-6x^3+11x^2-12x)$, where a is any nonzero real number, has zeros 0, 4, $1 \pm \sqrt{2}i$.

43. (a) $f(x) = a(x-1)(x+2)(x-2i)(x+2i)$

$= a(x-1)(x+2)(x^2+4)$

$f(-1) = 10 = a(-2)(1)(5) \Longrightarrow a = -1$

$f(x) = -(x-1)(x+2)(x-2i)(x+2i)$

(b) $f(x) = -(x-1)(x+2)(x^2+4)$

$= -(x^2+x-2)(x^2+4)$

$= -x^4-x^3-2x^2-4x+8$

44. (a) $f(x) = a(x+1)(-2)(x-i)(x+i)$

$= a(x+1)(x-2)(x^2+1)$

$f(1) = 8 = a(2)(-1)(2) \Longrightarrow a = -2$

$f(x) = -2(x+1)(x-2)(x-i)(x+i)$

(b) $f(x) = -2(x^2-x-2)(x^2+1)$

$= -2x^4+2x^3+2x^2+2x+4$

45. (a) $f(x) = a(x+1)\left(x-2-\sqrt{5}i\right)\left(x-2+\sqrt{5}i\right)$

$= a(x+1)(x^2-4x+4+5)$

$= a(x+1)(x^2-4x+9)$

$f(-2) = 42 = a(-1)(4+8+9) \Longrightarrow a = -2$

$f(x) = -2(x+1)\left(x-2-\sqrt{5}i\right)\left(x-2+\sqrt{5}i\right)$

(b) $f(x) = -2(x+1)(x^2-4x+9)$

$= -2x^3+6x^2-10x-18$

46. (a) $f(x) = a(x+2)\left(x-2-2\sqrt{2}i\right)\left(x-2+2\sqrt{2}i\right)$

$= a(x+2)(x^2-4x+4+8)$

$= a(x+2)(x^2-4x+12)$

$f(-1) = -34 = a(1)(17) \Longrightarrow a = -2$

$f(x) = -2(x+2)\left(x-2-2\sqrt{2}i\right)\left(x-2+2\sqrt{2}i\right)$

(b) $f(x) = -2(x+2)(x^2-4x+12)$

$= -2x^3+4x^2-8x-48$

47. $f(x) = x^4 - 6x^2 - 7$

(a) $f(x) = (x^2 - 7)(x^2 + 1)$

(b) $f(x) = (x - \sqrt{7})(x + \sqrt{7})(x^2 + 1)$

(c) $f(x) = (x - \sqrt{7})(x + \sqrt{7})(x + i)(x - i)$

48. $f(x) = x^4 + 6x^2 - 27$

(a) $f(x) = (x^2 + 9)(x^2 - 3)$

(b) $f(x) = (x^2 + 9)(x + \sqrt{3})(x - \sqrt{3})$

(c) $f(x) = (x + 3i)(x - 3i)(x + \sqrt{3})(x - \sqrt{3})$

49. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$

(a) $f(x) = (x^2 - 6)(x^2 - 2x + 3)$

(b) $f(x) = (x + \sqrt{6})(x - \sqrt{6})(x^2 - 2x + 3)$

(c) $f(x) = (x + \sqrt{6})(x - \sqrt{6})(x - 1 - \sqrt{2}i)(x - 1 + \sqrt{2}i)$

50. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$

(a) $f(x) = (x^2 + 4)(x^2 - 3x - 5)$

(b) $f(x) = (x^2 + 4)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

(c) $f(x) = (x + 2i)(x - 2i)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

51. $f(x) = 2x^3 + 3x^2 + 50x + 75$

Since $5i$ is a zero, so is $-5i$.

$$\begin{array}{r|cccc} 5i & 2 & 3 & 50 & 75 \\ & & 10i & -50 + 15i & -75 \\ \hline & 2 & 3 + 10i & 15i & 0 \end{array}$$

$$\begin{array}{r|ccc} -5i & 2 & 3 + 10i & 15i \\ & & -10i & -15i \\ \hline & 2 & 3 & 0 \end{array}$$

The zero of $2x + 3$ is $x = -\frac{3}{2}$. The zeros of f are $x = -\frac{3}{2}$ and $x = \pm 5i$.

Alternate Solution

Since $x = \pm 5i$ are zeros of $f(x)$, $(x + 5i)(x - 5i) = x^2 + 25$ is a factor of $f(x)$. By long division we have:

$$\begin{array}{r@{}l} & 2x + 3 \\ x^2 + 0x + 25 \,\big) & 2x^3 + 3x^2 + 50x + 75 \\ & \underline{2x^3 + 0x^2 + 50x} \\ & \qquad 3x^2 + 0x + 75 \\ & \qquad \underline{3x^2 + 0x + 75} \\ & \qquad\qquad 0 \end{array}$$

Thus, $f(x) = (x^2 + 25)(2x + 3)$ and the zeros of f are $x = \pm 5i$ and $x = -\frac{3}{2}$.

52. $f(x) = x^3 + x^2 + 9x + 9$

Since $3i$ is a zero, so is $-3i$.

$$\begin{array}{r|cccc} 3i & 1 & 1 & 9 & 9 \\ & & 3i & -9 + 3i & -9 \\ \hline & 1 & 1 + 3i & 3i & 0 \end{array}$$

$$\begin{array}{r|ccc} -3i & 1 & 1 + 3i & 3i \\ & & -3i & -3i \\ \hline & 1 & 1 & 0 \end{array}$$

The zeros of f are $3i$, $-3i$ and -1.

53. $g(x) = x^3 - 7x^2 - x + 87$. Since $5 + 2i$ is a zero, so is $5 - 2i$.

$$\begin{array}{r|cccc} 5 + 2i & 1 & -7 & -1 & 87 \\ & & 5 + 2i & -14 + 6i & -87 \\ \hline & 1 & -2 + 2i & -15 + 6i & 0 \end{array}$$

$$\begin{array}{r|ccc} 5 - 2i & 1 & -2 + 2i & -15 + 6i \\ & & 5 - 2i & 15 - 6i \\ \hline & 1 & 3 & 0 \end{array}$$

The zero of $x + 3$ is $x = -3$.

The zeros of f are $-3, 5 \pm 2i$.

54. $g(x) = 4x^3 + 23x^2 + 34x - 10$

Since $-3 + i$ is a zero, so is $-3 - i$.

$$\begin{array}{r|rrrr} -3+i & 4 & 23 & 34 & -10 \\ & & -12+4i & -37-i & 10 \\ \hline & 4 & 11+4i & -3-i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3-i & 4 & 11+4i & -3-i \\ & & -12-4i & 3+i \\ \hline & 4 & -1 & 0 \end{array}$$

The zero of $4x - 1$ is $x = \frac{1}{4}$. The zeros of $g(x)$ are $x = -3 \pm i$ and $x = \frac{1}{4}$.

Alternate Solution

Since $-3 \pm i$ are zeros of $g(x)$,

$$\begin{aligned}[x - (-3 + i)][x - (-3 - i)] &= [(x - 3) - i][(x + 3) + i] \\ &= (x + 3)^2 - i^2 = x^2 + 6x + 10\end{aligned}$$

is a factor of $g(x)$. By long division we have:

$$\begin{array}{r} 4x - 1 \\ x^2 + 6x + 10\overline{)\,4x^3 + 23x^2 + 34x - 10} \\ \underline{4x^3 + 24x^2 + 40x} \\ -x^2 - 6x - 10 \\ \underline{-x^2 - 6x - 10} \\ 0 \end{array}$$

Thus, $g(x) = (x^2 + 6x + 10)(4x - 1)$ and the zeros of g are $x = -3 \pm i$ and $x = \frac{1}{4}$.

55. $h(x) = 3x^3 - 4x^2 + 8x + 8$. Since $1 - \sqrt{3}i$ is a zero, so is $1 + \sqrt{3}i$.

$$\begin{array}{r|rrrr} 1-\sqrt{3}i & 3 & -4 & 8 & 8 \\ & & 3-3\sqrt{3}i & -10-2\sqrt{3}i & -8 \\ \hline & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1+\sqrt{3}i & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i \\ & & 3+3\sqrt{3}i & 2+2\sqrt{3}i \\ \hline & 3 & 2 & 0 \end{array}$$

The zero of $3x + 2$ is $x = -\frac{2}{3}$. The zeros of h are $x = -\frac{2}{3}, 1 \pm \sqrt{3}i$.

56. $f(x) = x^3 + 4x^2 + 14x + 20$

Since $-1 - 3i$ is a zero, so is $-1 + 3i$.

$$\begin{array}{r|rrrr} -1-3i & 1 & 4 & 14 & 20 \\ & & -1-3i & -12-6i & -20 \\ \hline & 1 & 3-3i & 2-6i & 0 \end{array}$$

$$\begin{array}{r|rrr} -1+3i & 1 & 3-3i & 2-6i \\ & & -1+3i & -2+6i \\ \hline & 1 & 2 & 0 \end{array}$$

The zero of $x + 2$ is $x = -2$. The zeros of f are $x = -2, -1 \pm 3i$.

57. $h(x) = 8x^3 - 14x^2 + 18x - 9$. Since $\frac{1}{2}(1 - \sqrt{5}i)$ is a zero, so is $\frac{1}{2}(1 + \sqrt{5}i)$.

$$\begin{array}{r|cccc}
\frac{1}{2}(1-\sqrt{5}i) & 8 & -14 & 18 & -9 \\
 & & 4 - 4\sqrt{5}i & -15 + 3\sqrt{5}i & 9 \\
\hline
 & 8 & -10 - 4\sqrt{5}i & 3 + 3\sqrt{5}i & 0
\end{array}$$

$$\begin{array}{r|ccc}
\frac{1}{2}(1+\sqrt{5}i) & 8 & -10 - 4\sqrt{5}i & 3 + 3\sqrt{5}i \\
 & & 4 + 4\sqrt{5}i & -3 - 3\sqrt{5}i \\
\hline
 & 8 & -6 & 0
\end{array}$$

The zero of $8x - 6$ is $x = \frac{3}{4}$. The zeros of h are $x = \frac{3}{4}, \frac{1}{2}(1 \pm \sqrt{5}i)$.

58. $f(x) = 25x^3 - 55x^2 - 54x - 18$

Since $\frac{1}{5}(-2 + \sqrt{2}i) = \frac{-2 + \sqrt{2}i}{5}$ is a zero, so is $\frac{-2 - \sqrt{2}i}{5}$.

$$\begin{array}{r|cccc}
\frac{-2+\sqrt{2}i}{5} & 25 & -55 & -54 & -18 \\
 & & -10 + 5\sqrt{2}i & 24 - 15\sqrt{2}i & 18 \\
\hline
\end{array}$$

$$\begin{array}{r|cccc}
\frac{-2-\sqrt{2}i}{5} & 25 & -65 + 5\sqrt{2}i & -30 - 15\sqrt{2}i & 0 \\
 & & -10 - 5\sqrt{2}i & 30 + 15\sqrt{2}i & \\
\hline
 & 25 & -75 & 0 &
\end{array}$$

The zero of $25x - 75$ is $x = 3$. The zeros of f are $x = 3, \frac{-2 \pm \sqrt{2}i}{5}$.

59. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$

(a) The root feature yields the real roots 1 and 2, and the complex roots $-3 \pm 1.414i$.

(b) By synthetic division:

$$\begin{array}{r|ccccc}
1 & 1 & 3 & -5 & -21 & 22 \\
 & & 1 & 4 & -1 & -22 \\
\hline
 & 1 & 4 & -1 & -22 & 0
\end{array}$$

$$\begin{array}{r|cccc}
2 & 1 & 4 & -1 & -22 \\
 & & 2 & 12 & 22 \\
\hline
 & 1 & 6 & 11 & 0
\end{array}$$

The complex roots of $x^2 + 6x + 11$ are $x = \dfrac{-6 \pm \sqrt{6^2 - 4(11)}}{2} = -3 \pm \sqrt{2}i$.

60. $f(x) = x^3 + 4x^2 + 14x + 20$

(a) Zeros: $-2, -1 \pm 3i$

(b) $x = -2$

$$\begin{array}{r|cccc}
-2 & 1 & 4 & 14 & 20 \\
 & & -2 & -4 & -20 \\
\hline
 & 1 & 2 & 10 & 0
\end{array}$$

$x^2 + 2x + 10$ has zeros $-1 \pm 3i$.

61. $h(x) = 8x^3 - 14x^2 + 18x - 9$

(a) The root feature yields the real root 0.75, and the complex roots $0.5 \pm 1.118i$.

(b) By synthetic division:

$$\begin{array}{r|cccc}
\frac{3}{4} & 8 & -14 & 18 & -9 \\
 & & 6 & -6 & 9 \\
\hline
 & 8 & -8 & 12 & 0
\end{array}$$

The complex roots of $8x^2 - 8x + 12$ are

$$x = \frac{8 \pm \sqrt{64 - 4(8)(12)}}{2(8)} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}i.$$

62. $f(x) = 25x^3 - 55x^2 - 54x - 18$

(a) Zeros: $3, -0.4 \pm 0.2828i$

(b)
$$\begin{array}{r|rrrr} 3 & 25 & -55 & -54 & -18 \\ & & 75 & 60 & 18 \\ \hline & 25 & 20 & 6 & 0 \end{array}$$

$25x^2 + 20x + 6$ has zeros $\dfrac{-2 \pm \sqrt{2}i}{5}$.

63. $-16t^2 + 48t = 64, \quad 0 \le t \le 3$

$-16t^2 + 48t - 64 = 0$

$$t = \frac{-48 \pm \sqrt{1792}i}{-32}$$

Since the roots are imaginary, the ball never will reach a height of 64 feet. You can verify this graphically by observing that $y_1 = -16t^2 + 48t$ and $y_2 = 64$ do not intersect.

64. No. Setting $P = R - C = xp - C = x(140 - 0.0001x) - (80x + 150{,}000) = 9{,}000{,}000$ yields a quadratic with no real roots.

$-0.0001x^2 + 60x - 9{,}150{,}000 = 0$

65. False, a third degree polynomial must have at least one real zero.

66. True. The complex conjugate of the zero $4 + 3i$ is also a zero.

67. $f(x) = x^4 - 4x^2 + k$

(a) f has two real zeros each of multiplicity 2 for $k = 4$: $f(x) = x^4 - 4x^2 + 4 = (x^2 - 2)^2$.

(b) f has two real zeros and two complex zeros if $k < 0$.

68. Answers will vary.

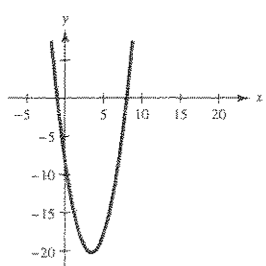
69. $f(x) = x^2 - 7x - 8 = \left(x^2 - 7x + \frac{49}{4}\right) - 8 - \frac{49}{4}$

$= \left(x - \frac{7}{2}\right)^2 - \frac{81}{4}$

Vertex: $\left(\frac{7}{2}, -\frac{81}{4}\right)$

$f(x) = (x - 8)(x + 1)$

Intercepts: $(8, 0), (-1, 0), (0, -8)$



70. $f(x) = -x^2 + x + 6$

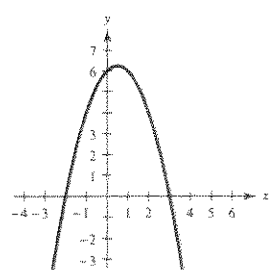
$= -\left(x^2 - x + \frac{1}{4}\right) + 6 + \frac{1}{4}$

$= -\left(x - \frac{1}{2}\right)^2 + \frac{25}{4}$

Vertex: $\left(\frac{1}{2}, \frac{25}{4}\right)$

$f(x) = -(x^2 - x - 6) = -(x - 3)(x + 2)$

Intercepts: $(3, 0), (-2, 0), (0, 6)$

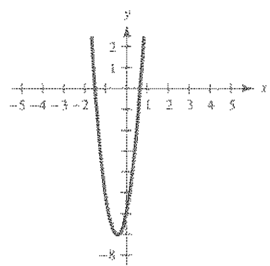


71. $f(x) = 6x^2 + 5x - 6 = (3x - 2)(2x + 3)$

Intercepts: $\left(\frac{2}{3}, 0\right), \left(-\frac{3}{2}, 0\right), (0, -6)$

$$\begin{aligned} f(x) &= 6x^2 + 5x - 6 \\ &= 6\left(x^2 + \tfrac{5}{6}x + \tfrac{25}{144}\right) - 6 - \tfrac{25}{24} \\ &= 6\left(x + \tfrac{5}{12}\right)^2 - \tfrac{169}{24} \end{aligned}$$

Vertex: $\left(-\frac{5}{12}, -\frac{169}{24}\right)$

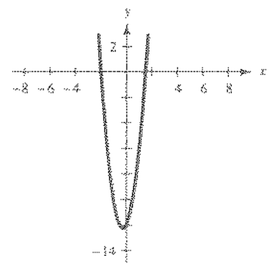


72.
$$\begin{aligned} f(x) &= 4x^2 + 2x - 12 \\ &= 4\left(x^2 + \tfrac{1}{2}x + \tfrac{1}{16}\right) - 12 - \tfrac{1}{4} \\ &= 4\left(x + \tfrac{1}{4}\right)^2 - \tfrac{49}{4} \end{aligned}$$

Vertex: $\left(-\frac{1}{4}, -\frac{49}{4}\right)$

$f(x) = (2x - 3)(2x + 4)$

Intercepts: $\left(\frac{3}{2}, 0\right), (-2, 0), (0, -12)$



Section 2.6 Rational Functions and Asymptotes

- You should know the following basic facts about rational functions.
  (a) A function of the form $f(x) = P(x)/Q(x)$, $Q(x) \neq 0$, where $P(x)$ and $Q(x)$ are polynomials, is called a rational function.
  (b) The domain of a rational function is the set of all real numbers except those which make the denominator zero.
  (c) If $f(x) = P(x)/Q(x)$ is in reduced form, and a is a value such that $Q(a) = 0$, then the line $x = a$ is a vertical asymptote of the graph of f. $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$.
  (d) The line $y = b$ is a horizontal asymptote of the graph of f if $f(x) \to b$ as $x \to \infty$ or $x \to -\infty$.
  (e) Let $f(x) = \dfrac{P(x)}{Q(x)} = \dfrac{a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0}{b_mx^m + b_{m-1}x^{m-1} + \cdots + b_1x + b_0}$ where $P(x)$ and $Q(x)$ have no common factors.
    1. If $n < m$, then the x-axis ($y = 0$) is a horizontal asymptote.
    2. If $n = m$, then $y = \dfrac{a_n}{b_m}$ is a horizontal asymptote.
    3. If $n > m$, then there are no horizontal asymptotes.

Vocabulary Check

1. rational functions **2.** vertical asymptote **3.** horizontal asymptote

1. $f(x) = \dfrac{1}{x-1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	-2
0.9	-10
0.99	-100
0.999	-1000

x	$f(x)$
1.5	2
1.1	10
1.01	100
1.001	1000

x	$f(x)$
5	0.25
10	$0.\overline{1}$
100	$0.\overline{01}$
1000	$0.\overline{001}$

x	$f(x)$
-5	$-0.1\overline{6}$
-10	$-0.\overline{09}$
-100	$-0.\overline{0099}$
-1000	$-0.\overline{000999}$

(c) f approaches $-\infty$ from the left of 1 and ∞ from the right of 1.

2. $f(x) = \dfrac{5x}{x-1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	-5
0.9	-45
0.99	-495
0.999	-4995

x	$f(x)$
1.5	15
1.1	55
1.01	505
1.001	5005

x	$f(x)$
5	6.25
10	$5.\overline{55}$
100	$5.\overline{05}$
1000	$5.\overline{005}$

x	$f(x)$
-5	$4.1\overline{6}$
-10	$4.\overline{54}$
-100	$4.95\overline{0495}$
-1000	4.995

(c) f approaches $-\infty$ from the left of 1 and ∞ from the right of 1.

3. $f(x) = \dfrac{3x}{|x-1|}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	3
0.9	27
0.99	297
0.999	2997

x	$f(x)$
1.5	9
1.1	33
1.01	303
1.001	3003

x	$f(x)$
5	3.75
10	$3.\overline{33}$
100	$3.\overline{03}$
1000	$3.\overline{003}$

x	$f(x)$
-5	-2.5
-10	-2.727
-100	-2.970
-1000	-2.997

(c) f approaches ∞ from both the left and the right of 1.

4. $f(x) = \dfrac{3}{|x-1|}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	6
0.9	30
0.99	300
0.999	3000

x	$f(x)$
1.5	6
1.1	30
1.01	300
1.001	3000

x	$f(x)$
5	0.75
10	$0.\overline{33}$
100	$0.\overline{03}$
1000	$0.\overline{003}$

x	$f(x)$
-5	0.5
-10	$0.\overline{27}$
-100	0.0297
-1000	0.003

(c) f approaches ∞ from both the left and the right of 1.

5. $f(x) = \dfrac{3x^2}{x^2 - 1}$

(a) Domain: all $x \neq \pm 1$

(b)

x	$f(x)$
0.5	-1
0.9	-12.79
0.99	-147.8
0.999	-1498

x	$f(x)$
1.5	5.4
1.1	17.29
1.01	152.3
1.001	1502.3

x	$f(x)$
5	3.125
10	$3.\overline{03}$
100	$3.\overline{0003}$
1000	3

x	$f(x)$
-5	3.125
-10	$3.\overline{03}$
-100	$3.\overline{0003}$
-1000	3

(c) f approaches $-\infty$ from the left of 1, and ∞ from the right of 1. f approaches ∞ from the left of -1, and $-\infty$ from the right of -1.

6. $f(x) = \dfrac{4x}{x^2 - 1}$

(a) Domain: all $x \neq \pm 1$

(b)

x	$f(x)$
0.5	$-2.\overline{66}$
0.9	-18.95
0.99	-199
0.999	-1999

x	$f(x)$
1.5	4.8
1.1	20.95
1.01	201
1.001	2001

x	$f(x)$
5	$-0.8\overline{33}$
10	$0.\overline{40}$
100	0.04
1000	0.004

x	$f(x)$
-5	$-0.8\overline{33}$
-10	$-0.\overline{40}$
-100	-0.04
-1000	-0.004

(c) f approaches $-\infty$ from the left of 1, and ∞ from the right of 1. f approaches $-\infty$ from the left of -1, and ∞ from the right of -1.

7. $f(x) = \dfrac{2}{x + 2}$

Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 0$

Matches graph (a).

8. $f(x) = \dfrac{1}{x - 3}$

Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$

Matches graph (d).

9. $f(x) = \dfrac{4x + 1}{x}$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 4$

Matches graph (c).

10. $f(x) = \dfrac{1 - x}{x}$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = -1$

Matches graph (e).

11. $f(x) = \dfrac{x - 2}{x - 4}$

Vertical asymptote: $x = 4$

Horizontal asymptote: $y = 1$

Matches graph (b).

12. $f(x) = -\dfrac{x + 2}{x + 4}$

Vertical asymptote: $x = -4$

Horizontal asymptote: $y = -1$

Matches graph (f).

13. $f(x) = \dfrac{1}{x^2}$

(a) Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

(b) Holes: none

14. $f(x) = \dfrac{3}{(x - 2)^3}$

(a) Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

(b) Holes: none

15. $f(x) = \dfrac{x(2 + x)}{2x - x^2} = \dfrac{2 + x}{2 - x}, x \neq 0$

(a) Vertical asymptote: $x = 2$

Horizontal asymptote: $y = -1$

(b) Hole at $x = 0$: $(0, 1)$

16. $f(x) = \dfrac{x^2 + 2x + 1}{2x^2 - x - 3} = \dfrac{(x + 1)^2}{(x + 1)(2x - 3)} = \dfrac{x + 1}{2x - 3},$ $x \neq -1$

(a) Vertical asymptote: $x = \dfrac{3}{2}$

Horizontal asymptote: $y = \dfrac{1}{2}$

(b) Hole at $x = -1$: $(-1, 0)$

17. $f(x) = \dfrac{x^2 - 25}{x^2 + 5x}$

$= \dfrac{(x - 5)(x + 5)}{x(x + 5)}$

$= \dfrac{x - 5}{x}, \; x \neq -5$

(a) Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 1$

(b) Hole at $x = -5$: $(-5, 2)$

18. $f(x) = \dfrac{-(5x^2 + 14x - 3)}{2x^2 + 7x + 3}$

$= \dfrac{-(x + 3)(5x - 1)}{(x + 3)(2x + 1)}$

$= -\dfrac{5x - 1}{2x + 1}, \; x \neq -3$

(a) Vertical asymptote: $x = -\dfrac{1}{2}$

Horizontal asymptote: $y = -\dfrac{5}{2}$

(b) Hole at $x = -3$: $\left(-3, -\dfrac{16}{5}\right)$

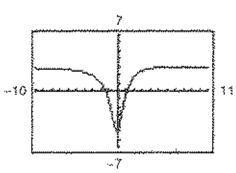
19. $f(x) = \dfrac{3x^2 + x - 5}{x^2 + 1}$

(a) Domain: all real numbers

(b) Vertical asymptote: none

Horizontal asymptote: $y = 3$

(c)



20. $f(x) = \dfrac{3x^2 + 1}{x^2 + x + 9}$

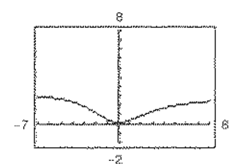
(a) Domain: All real numbers. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]

(b) Vertical asymptote: none

Horizontal asymptote: $y = 3$

[degree $p(x)$ = degree $q(x)$]

(c)



21. $f(x) = \dfrac{x - 3}{|x|}$

(a) Domain: all real numbers except $x = 0$

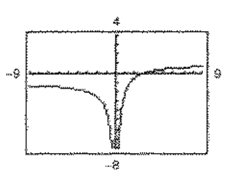
(b) Vertical asymptote: $x = 0$

Horizontal asymptote:

$y = 1$ to the right

$y = -1$ (to the left)

(c)



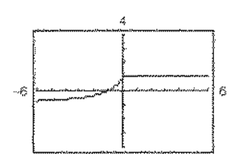
22. $f(x) = \dfrac{x + 1}{|x| + 1}$

(a) Domain: all x

(b) No vertical asymptotes.

Horizontal asymptotes $y = \pm 1$

(c)



23. $f(x) = \dfrac{x^2 - 16}{x - 4}$, $g(x) = x + 4$

(a) Domain of f: all real numbers except 4

Domain of g: all real numbers

(b) $f(x) = \dfrac{(x - 4)(x + 4)}{x - 4} = x + 4,\ x \neq 4$

f has no vertical asymptotes.

(c) Hole at $x = 4$

(d)

x	1	2	3	4	5	6	7
$f(x)$	5	6	7	Undef.	9	10	11
$g(x)$	5	6	7	8	9	20	11

(e) f and g differ at $x = 4$, where f is undefined.

24. $f(x) = \dfrac{x^2 - 9}{x - 3}$, $g(x) = x + 3$

(a) Domain of f: all real numbers except 3

Domain of g: all real numbers

(b) $f(x) = \dfrac{(x - 3)(x + 3)}{x - 3} = x + 3,\ x \neq 3$

f has no vertical asymptotes.

(c) Hole at $x = 3$

(d)

x	0	1	2	3	4	5	6
$f(x)$	3	4	5	Undef.	7	8	9
$g(x)$	3	4	5	6	7	8	9

(e) f and g differ at $x = 3$, where f is undefined.

25. $f(x) = \dfrac{x^2 - 1}{x^2 - 2x - 3} = \dfrac{(x - 1)(x + 1)}{(x + 1)(x - 3)}$, $g(x) = \dfrac{x - 1}{x - 3}$

(a) Domain of f: all real numbers except -1, 3

Domain of g: all real numbers except 3

(b) $f(x) = \dfrac{(x - 1)(x + 1)}{(x + 1)(x - 3)} = \dfrac{x - 1}{x - 3},\ x \neq -1$

f has a vertical asymptote at $x = 3$.

(c) The graph has a hole at $x = -1$.

(d)

x	-2	-1	0	1	2	3	4
$f(x)$	$\frac{3}{5}$	Undef.	$\frac{1}{3}$	0	-1	Undef.	3
$g(x)$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	0	-1	Undef.	3

(e) f and g differ at $x = -1$, where f is undefined.

26. $f(x) = \dfrac{x^2 - 4}{x^2 - 3x + 2} = \dfrac{(x + 2)(x - 2)}{(x - 2)(x - 1)}$, $g(x) = \dfrac{x + 2}{x - 1}$

(a) Domain of f: all real numbers except 1 and 2

Domain of g: all real numbers except 1

(b) $f(x) = \dfrac{(x + 2)(x - 2)}{(x - 2)(x - 1)} = \dfrac{x + 2}{x - 1},\ x \neq 2$

f has a vertical asymptote at $x = 1$.

(c) The graph has a hole at $x = 2$.

(d)

x	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{4}$	0	$-\frac{1}{2}$	-2	Undef.	Undef.	3
$g(x)$	$\frac{1}{4}$	0	$-\frac{1}{2}$	-2	Undef.	4	3

(e) f and g differ at $x = 2$, where f is undefined.

27. $f(x) = 4 - \dfrac{1}{x}$

(a) As $x \to \pm\infty, f(x) \to 4$

(b) As $x \to \infty, f(x) \to 4$ but is less than 4.

(c) As $x \to -\infty, f(x) \to 4$ but is greater than 4.

28. $f(x) = 2 + \dfrac{1}{x - 3}$

(a) As $x \to \pm\infty, f(x) \to 2$.

(b) As $x \to \infty, f(x) \to 2$ but is greater than 2.

(c) As $x \to -\infty, f(x) \to 2$ but is less than 2.

29. $f(x) = \dfrac{2x - 1}{x - 3}$

(a) As $x \to \pm\infty, f(x) \to 2$.

(b) As $x \to \infty, f(x) \to 2$ but is greater than 2.

(c) As $x \to -\infty, f(x) \to 2$ but is less than 2.

30. $f(x) = \dfrac{2x - 1}{x^2 + 1}$

(a) As $x \to \pm\infty, f(x) \to 0$.

(b) As $x \to \infty, f(x) \to 0$ but is greater than 0.

(c) As $x \to -\infty, f(x) \to 0$ but is less than 0.

31. $g(x) = \dfrac{x^2 - 4}{x + 3} = \dfrac{(x - 2)(x + 2)}{x + 3}$

The zeros of g are the zeros of the numerator: $x = \pm 2$

32. $g(x) = \dfrac{x^3 - 8}{x^2 + 4}$

The zero of g corresponds to the zero of the numerator and is $x = 2$.

33. $f(x) = 1 - \dfrac{2}{x - 5} = \dfrac{x - 7}{x - 5}$

The zero of f corresponds to the zero of the numerator and is $x = 7$.

34. $h(x) = 5 + \dfrac{3}{x^2 + 1}$

There are no real zeros.

35. $g(x) = \dfrac{x^2 - 2x - 3}{x^2 + 1} = \dfrac{(x - 3)(x + 1)}{x^2 + 1} = 0$

Zeros: $x = -1, 3$

36. $g(x) = \dfrac{x^2 - 5x + 6}{x^2 + 4} = \dfrac{(x - 3)(x - 2)}{x^2 + 4} = 0$

Zeros: $x = 2, 3$

37. $f(x) = \dfrac{2x^2 - 5x + 2}{2x^2 - 7x + 3} = \dfrac{(2x - 1)(x - 2)}{(2x - 1)(x - 3)} = \dfrac{x - 2}{x - 3}$, $x \neq \dfrac{1}{2}$

Zero: $x = 2$ $\left(x = \dfrac{1}{2} \text{ is not in the domain.}\right)$

38. $f(x) = \dfrac{2x^2 + 3x - 2}{x^2 + x - 2} = \dfrac{(x + 2)(2x - 1)}{(x + 2)(x + 1)} = \dfrac{2x - 1}{x + 1}$, $x = -2$

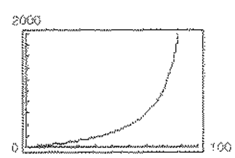
Zero: $x = \dfrac{1}{2}$ ($x = -2$ is not in the domain.)

39. $C = \dfrac{255p}{100 - p}$, $0 \leq p < 100$

(a) $C(10) = \dfrac{255(10)}{100 - 10} \approx 28.33$ million dollars

(b) $C(40) = \dfrac{255(40)}{100 - 40} = 170$ million dollars

(c) $C(75) = \dfrac{255(75)}{100 - 75} = 765$ million dollars

(d) 

(e) $C \to \infty$ as $x \to 100$. No, it would not be possible to remove 100% of the pollutants.

40. (a) $C = \dfrac{25{,}000(15)}{100 - 15} \approx 4411.76$

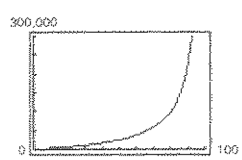
The cost would be $4411.76.

(b) $C = \dfrac{25{,}000(50)}{100 - 50} = 25{,}000$

The cost would be $25,000.

(c) $C = \dfrac{25{,}000(90)}{100 - 90} = 225{,}000$

The cost would be $225,000.

(d) 

(e) No. The model is undefined for $p = 100$.

41. (a) Use data $\left(16, \frac{1}{3}\right), \left(32, \frac{1}{4.7}\right), \left(44, \frac{1}{9.8}\right),$

$\left(50, \frac{1}{19.7}\right), \left(60, \frac{1}{39.4}\right).$

$$\frac{1}{y} = -0.007x + 0.445$$

$$y = \frac{1}{0.445 - 0.007x}$$

(b)

x	16	32	44	50	60
y	3.0	4.5	7.3	10.5	40

(Answers will vary.)

(c) No, the function is negative for $x = 70$.

42. (a)

M	200	400	600	800	1000	1200	1400	1600	1800	2000
t	0.472	0.596	0.710	0.817	0.916	1.009	1.096	1.178	1.255	1.328

The greater the mass, the more time required per oscillation. The model is a good fit to the actual data.

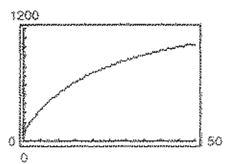
(b) You can find M corresponding to $t = 1.056$ by finding the point of intersection of

$$t = \frac{38M + 16{,}965}{10(M + 500)} \text{ and } t = 1.056.$$

If you do this, you obtain $M \approx 1306$ grams.

43. $N = \dfrac{20(5 + 3t)}{1 + 0.04t}, \; 0 \le t$

(a)



(b) $N(5) \approx 333$ deer

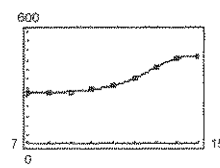
$N(10) = 500$ deer

$N(25) = 800$ deer

(c) The herd is limited by the horizontal asymptote:

$$N = \frac{60}{0.04} = 1500 \text{ deer}$$

44. (a)



The model is a good fit.

(b) For 2010, $t = 20$ and $D \approx \$366.8$ billion.

For 2015, $t = 25$ and $D \approx \$332.3$ billion.

For 2020, $t = 30$ and $D \approx \$319.1$ billion.

Answers will vary.

(c) Horizontal asymptote

$$y = \frac{1.493}{0.0051} \approx 292.7$$

As time passes, the national defense outlays approach \$292.7 billion.

45. False. A rational function can have at most n vertical asymptotes, where n is the degree of the denominator.

46. False. For example, $f(x) = \dfrac{1}{x^2 + 1}$ has no vertical asymptote.

47. There are vertical asymptotes at $x = \pm 3$, and zeros at $x = \pm 2$. Matches (b).

48. There are vertical asymptotes at $x = \pm 1$, and $x = 0$ is a zero. Matches (c).

49. $f(x) = \dfrac{x - 1}{x^3 - 8}$

50. $f(x) = \dfrac{x - 2}{(x + 1)^2}$

51. $f(x) = \dfrac{2(x + 3)(x - 3)}{(x + 2)(x - 1)} = \dfrac{2x^2 - 18}{x^2 + x - 2}$

52. $f(x) = \dfrac{-2(x + 2)(x - 3)}{(x + 1)(x - 2)} = \dfrac{-2x^2 + 2x + 12}{x^2 - x - 2}$

53. $y - 2 = \dfrac{-1-2}{0-3}(x-3) = 1(x-3)$

$y = x - 1$

$y - x + 1 = 0$

54. $y - 1 = \dfrac{1+5}{-6-4}(x+6)$

$-10y + 10 = 6x + 36$

$3x + 5y + 13 = 0$

55. $y - 7 = \dfrac{10-7}{3-2}(x-2) = 3(x-2)$

$y = 3x + 1$

$3x - y + 1 = 0$

56. $y - 0 = \dfrac{4-0}{-9-0}(x-0)$

$-9y = 4x$

$4x + 9y = 0$

57.

$$\begin{array}{r}
x + 9 \\
x - 4 \overline{\smash{\big)}\, x^2 + 5x + 6} \\
\underline{x^2 - 4x} \phantom{{}+6} \\
9x + 6 \\
\underline{9x - 36} \\
42
\end{array}$$

$$\frac{x^2 + 5x + 6}{x - 4} = x + 9 + \frac{42}{x - 4}$$

58.

$$\begin{array}{r|rrr}
3 & 1 & -10 & 15 \\
 & & 3 & -21 \\ \hline
 & 1 & -7 & -6
\end{array}$$

$$\frac{x^2 - 10x + 15}{x - 3} = x - 7 + \frac{-6}{x - 3}$$

59.

$$\begin{array}{r}
2x^2 \phantom{{}+0x+{}} - 9 \\
x^2 + 5 \overline{\smash{\big)}\, 2x^4 + 0x^3 + x^2 + 0x - 11} \\
\underline{2x^4 + \phantom{0x^3+{}} 10x^2 \phantom{{}+0x-11}} \\
-9x^2 - 11 \\
\underline{-9x^2 - 45} \\
34
\end{array}$$

$$\frac{2x^4 + x^2 - 11}{x^2 + 5} = 2x^2 - 9 + \frac{34}{x^2 + 5}$$

60.

$$\begin{array}{r}
2x^4 - 3x^3 + 6x^2 - 9x + \tfrac{27}{2} \\
2x + 3 \overline{\smash{\big)}\, 4x^5 + 0x^4 + 3x^3 + 0x^2 + 0x - 10} \\
\underline{4x^5 + 6x^4} \phantom{{}+3x^3+0x^2+0x-10} \\
-6x^4 + 3x^3 \phantom{{}+0x^2+0x-10} \\
\underline{-6x^4 - 9x^3} \phantom{{}+0x^2+0x-10} \\
12x^3 \phantom{{}+0x^2+0x-10} \\
\underline{12x^3 + 18x^2} \phantom{{}+0x-10} \\
-18x^2 \phantom{{}+0x-10} \\
\underline{-18x^2 - 27x} \phantom{{}-10} \\
27x - 10 \\
\underline{27x + \tfrac{81}{2}} \\
-\tfrac{101}{2}
\end{array}$$

$$\frac{4x^5 + 3x^3 - 10}{2x + 3} = 2x^4 - 3x^3 + 6x^2 - 9x + \frac{27}{2} - \frac{101}{4x + 6}$$

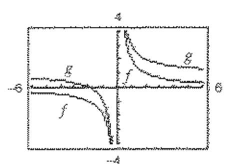
Section 2.7 Graphs of Rational Functions

- You should be able to graph $f(x) = \dfrac{p(x)}{q(x)}$.
  - (a) Find the x- and y-intercepts.
  - (b) Find any vertical or horizontal asymptotes.
  - (c) Plot additional points.
  - (d) If the degree of the numerator is one more than the degree of the denominator, use long division to find the slant asymptote.

Vocabulary Check

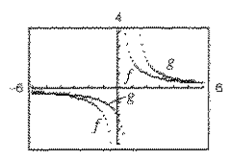
1. slant, asymptote
2. vertical

1. $g(x) = \dfrac{2}{x} + 1$



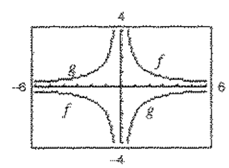
Vertical shift one unit upward

2.



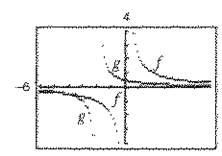
Horizontal shift one unit to the right

3. $g(x) = -\dfrac{2}{x}$



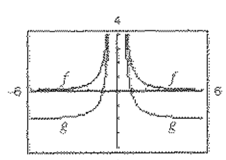
Reflection in the x-axis

4.



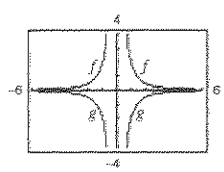
Horizontal shift two units to the left, and vertical shrink

5. $g(x) = \dfrac{2}{x^2} - 2$



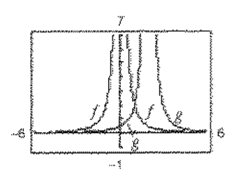
Vertical shift two units downward

6.



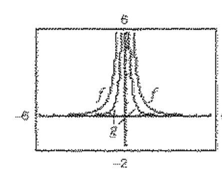
Reflection in the x-axis

7. $g(x) = \dfrac{2}{(x-2)^2}$



Horizontal shift two units to the right

8.



Each y-value is multiplied by $\frac{1}{4}$.

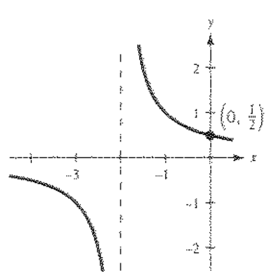
Vertical shrink

9. $f(x) = \dfrac{1}{x+2}$

y-intercept: $\left(0, \dfrac{1}{2}\right)$

Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 0$



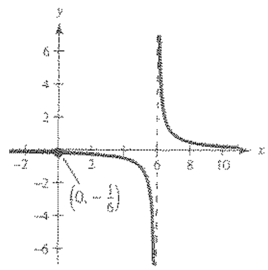
x	-4	-3	-1	0	1
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$

10. $f(x) = \dfrac{1}{x-6}$

y-intercept: $\left(0, -\dfrac{1}{6}\right)$

Vertical asymptote: $x = 6$

Horizontal asymptote: $y = 0$



x	-1	0	2	4	8	10
y	$-\frac{1}{7}$	$-\frac{1}{6}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$

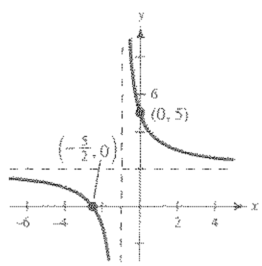
11. $C(x) = \dfrac{5+2x}{1+x} = \dfrac{2x+5}{x+1}$

x-intercept: $\left(-\dfrac{5}{2}, 0\right)$

y-intercept: $(0, 5)$

Vertical asymptote: $x = -1$

Horizontal asymptote: $y = 2$



x	-4	-3	-2	0	1	2
$C(x)$	1	$\frac{1}{2}$	-1	5	$\frac{7}{2}$	3

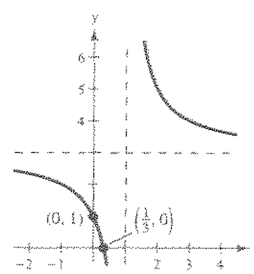
12. $P(x) = \dfrac{1-3x}{1-x} = \dfrac{3x-1}{x-1}$

x-intercept: $\left(\dfrac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 3$



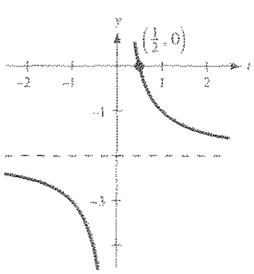
x	-1	0	2	3
y	2	1	5	4

13. $f(t) = \dfrac{1-2t}{t} = -\dfrac{2t-1}{t}$

t-intercept: $\left(\dfrac{1}{2}, 0\right)$

Vertical asymptote: $t = 0$

Horizontal asymptote: $y = -2$



x	-2	-1	$\frac{1}{2}$	1	2
y	$-\frac{5}{2}$	-3	0	-1	$-\frac{3}{2}$

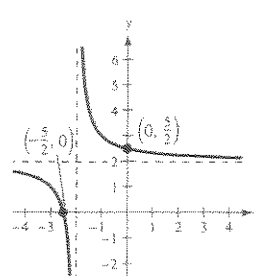
14. $g(x) = \dfrac{1}{x+2} + 2 = \dfrac{2x+5}{x+2}$

y-intercept: $\left(0, \dfrac{5}{2}\right)$

x-intercept: $\left(-\dfrac{5}{2}, 0\right)$

Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 2$



x	-4	$-\frac{5}{2}$	-1	0	2
y	$\frac{3}{2}$	0	3	$\frac{5}{2}$	$\frac{9}{4}$

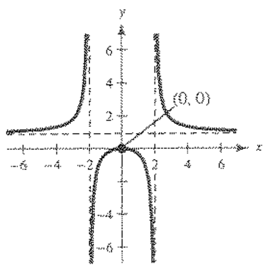
15. $f(x) = \dfrac{x^2}{x^2 - 4}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = 2,\ x = -2$

Horizontal asymptote: $y = 1$

y-axis symmetry



x	-4	-1	0	-1	4
y	$\frac{4}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{4}{3}$

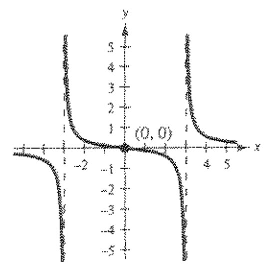
16. $g(x) = \dfrac{x}{x^2 - 9}$

Intercepts: $(0, 0)$

Vertical asymptotes: $x = \pm 3$

Horizontal asymptote: $y = 0$

Origin symmetry



x	-5	-4	-2	0	2	4	5
y	$-\frac{5}{16}$	$-\frac{4}{7}$	$\frac{2}{5}$	0	$-\frac{2}{5}$	$\frac{4}{7}$	$\frac{5}{16}$

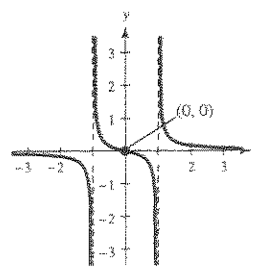
17. $f(x) = \dfrac{x}{x^2 - 1} = \dfrac{x}{(x + 1)(x - 1)}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = 1$ and $x = -1$

Horizontal asymptote: $y = 0$

Origin symmetry



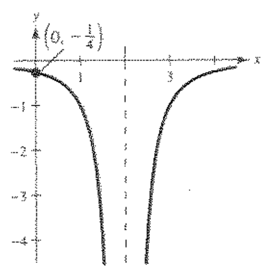
x	-3	-2	$-\frac{1}{2}$	0	$\frac{1}{2}$	2	3	4
y	$-\frac{3}{8}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	$-\frac{2}{3}$	$\frac{2}{3}$	$\frac{3}{8}$	$\frac{4}{15}$

18. $f(x) = -\dfrac{1}{(x - 2)^2}$

y-intercept: $\left(0, -\dfrac{1}{4}\right)$

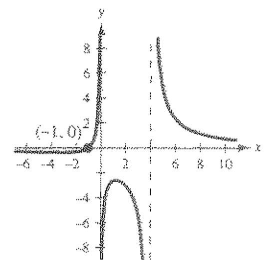
Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$



x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}$	3	$\frac{7}{2}$	4
y	$-\frac{1}{4}$	$-\frac{4}{9}$	-1	-4	-4	-1	$-\frac{4}{9}$	$-\frac{1}{4}$

19. $g(x) = \dfrac{4(x+1)}{x(x-4)}$



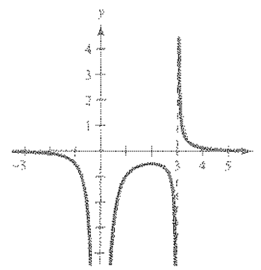
Intercept: $(-1, 0)$

Vertical asymptotes: $x = 0$ and $x = 4$

Horizontal asymptote: $y = 0$

x	-2	-1	1	2	3	5	6
y	$-\frac{1}{3}$	0	$-\frac{8}{3}$	-3	$-\frac{16}{3}$	$\frac{24}{5}$	$\frac{7}{3}$

20. $h(x) = \dfrac{2}{x^2(x-3)}$

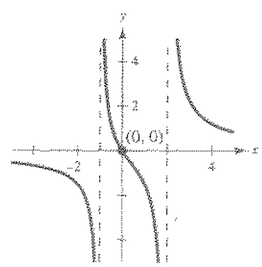


Vertical asymptotes: $x = 0, x = 3$

Horizontal asymptote: $y = 0$

x	-2	0	1	2	3	4
y	$-\frac{1}{10}$	Undef.	-1	$-\frac{1}{2}$	Undef.	$\frac{1}{8}$

21. $f(x) = \dfrac{3x}{x^2 - x - 2} = \dfrac{3x}{(x+1)(x-2)}$



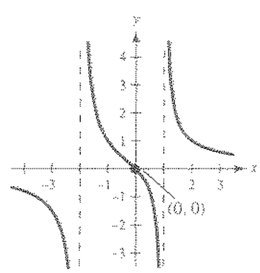
Intercept: $(0, 0)$

Vertical asymptotes: $x = -1, 2$

Horizontal asymptote: $y = 0$

x	-3	0	1	3	4
y	$-\frac{9}{10}$	0	$-\frac{3}{2}$	$\frac{9}{4}$	$\frac{6}{5}$

22. $f(x) = \dfrac{2x}{x^2 + x - 2} = \dfrac{2x}{(x+2)(x-1)}$



Intercept: $(0, 0)$

Vertical asymptotes: $x = -2, 1$

Horizontal asymptote: $y = 0$

x	-4	-3	-1	0	$\frac{1}{2}$	2	3
y	$-\frac{4}{5}$	$-\frac{3}{2}$	1	0	$-\frac{4}{5}$	1	$\frac{3}{5}$

23. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6} = \dfrac{x(x+3)}{(x-2)(x+3)} = \dfrac{x}{x-2}$,

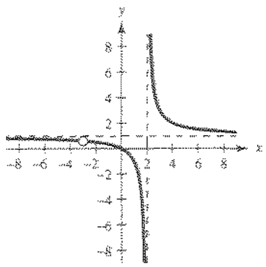
$x \neq -3$

Intercept: $(0, 0)$

Vertical asymptote: $x = 2$

(There is a hole at $x = -3$.)

Horizontal asymptote: $y = 1$



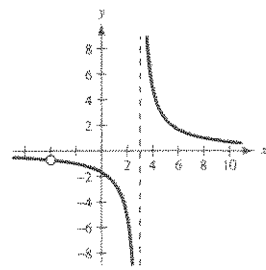
x	-2	-1	0	1	2	3
y	$\frac{1}{2}$	$\frac{1}{3}$	0	-1	Undef.	3

24. $g(x) = \dfrac{5(x+4)}{x^2 + x - 12} = \dfrac{5(x+4)}{(x+4)(x-3)} = \dfrac{5}{x-3}$,

$x \neq -4$

Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$



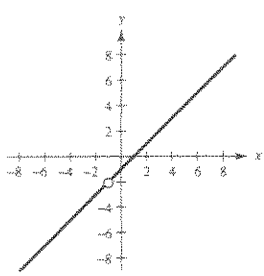
Hole at $x = -4$

x	-4	0	1	3	4
y	Undef.	$-\frac{5}{3}$	$-\frac{5}{2}$	Undef.	5

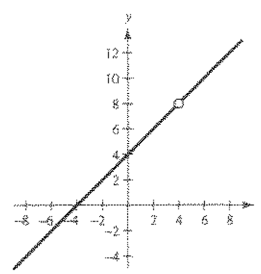
25. $f(x) = \dfrac{x^2 - 1}{x + 1} = \dfrac{(x+1)(x-1)}{x+1} = x - 1$,

$x \neq -1$

The graph is a line, with a hole at $x = -1$.



26. $f(x) = \dfrac{x^2 - 16}{x - 4} = x + 4, x \neq 4$

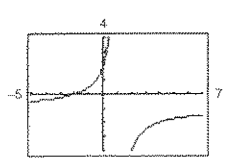


Hole at $x = 4$

27. $f(x) = \dfrac{2 + x}{1 - x} = -\dfrac{x + 2}{x - 1}$

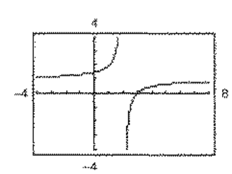
Vertical asymptote: $x = 1$

Horizontal asymptote: $y = -1$



Domain: $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

28. $f(x) = \dfrac{3 - x}{2 - x} = \dfrac{x - 3}{x - 2}$



x-intercept: $(3, 0)$

y-intercept: $\left(0, \dfrac{3}{2}\right)$

Vertical asymptote: $x = 2$

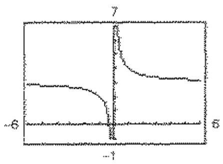
Horizontal asymptote: $y = 1$

Domain: all $x \neq 2$

29. $f(t) = \dfrac{3t + 1}{t}$

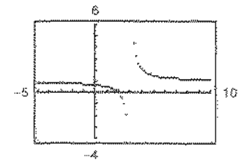
Vertical asymptote: $t = 0$

Horizontal asymptote: $y = 3$



Domain: $t \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

30. $h(x) = \dfrac{x - 2}{x - 3}$



x-intercept: $(2, 0)$

y-intercept: $\left(0, \dfrac{2}{3}\right)$

Vertical asymptote: $x = 3$

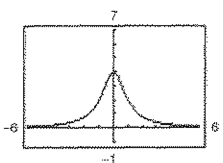
Horizontal asymptote: $y = 1$

Domain: all $x \neq 3$

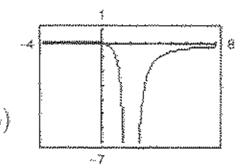
31. $h(t) = \dfrac{4}{t^2 + 1}$

Domain: all real numbers OR $(-\infty, \infty)$

Horizontal asymptote: $y = 0$



32. $g(x) = -\dfrac{x}{(x - 2)^2}$



Domain: all real numbers except 2 or $(-\infty, 2) \cup (2, \infty)$

Vertical asymptote: $x = 2$

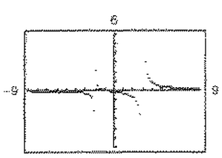
Horizontal asymptote: $y = 0$

33. $f(x) = \dfrac{x + 1}{x^2 - x - 6} = \dfrac{x + 1}{(x - 3)(x + 2)}$

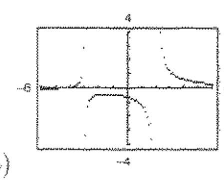
Domain: all real numbers except $x = 3, -2$

Vertical asymptotes: $x = 3,\ x = -2$

Horizontal asymptote: $y = 0$



34. $f(x) = \dfrac{x + 4}{x^2 + x - 6}$



Domain: all real numbers except -3 and 2 or $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

Vertical asymptotes: $x = -3, x = 2$

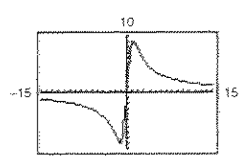
Horizontal asymptote: $y = 0$

35. $f(x) = \dfrac{20x}{x^2 + 1} - \dfrac{1}{x} = \dfrac{19x^2 - 1}{x(x^2 + 1)}$

Domain: all real numbers except 0, OR $(-\infty, 0) \cup (0, \infty)$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

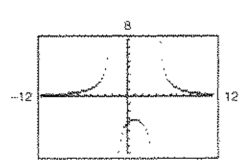


36. $f(x) = 5\left(\dfrac{1}{x - 4} - \dfrac{1}{x + 2}\right) = \dfrac{30}{(x - 4)(x + 2)}$

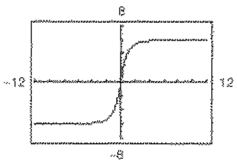
Domain: all real numbers except -2 and 4

Vertical asymptotes: $x = -2, x = 4$

Horizontal asymptote: $y = 0$

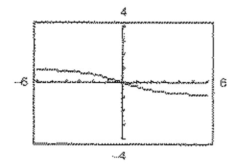


37. $h(x) = \dfrac{6x}{\sqrt{x^2+1}}$



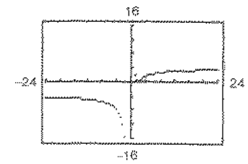
There are two horizontal asymptotes, $y = \pm 6$.

38. $f(x) = \dfrac{-x}{\sqrt{9+x^2}}$



There are two horizontal asymptotes, $y = \pm 1$.

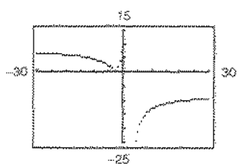
39. $g(x) = \dfrac{4|x-2|}{x+1}$



There are two horizontal asymptotes, $y = \pm 4$.

One vertical asymptote: $x = -1$

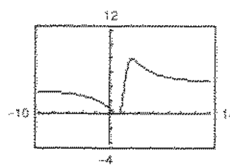
40. $f(x) = \dfrac{-8|3+x|}{x-2} = \dfrac{8|3+x|}{2-x}$



There are two horizontal asymptotes, $y = -8$ and $y = 8$.

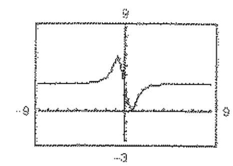
Vertical asymptote: $x = 2$

41. $f(x) = \dfrac{4(x-1)^2}{x^2-4x+5}$



The graph crosses its horizontal asymptote, $y = 4$.

42. $g(x) = \dfrac{3x^4-5x+3}{x^4+1}$



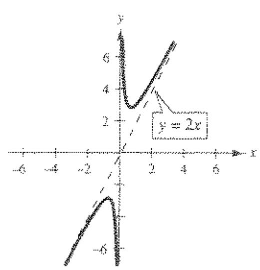
The graph crosses its horizontal asymptote, $y = 3$.

43. $f(x) = \dfrac{2x^2+1}{x} = 2x + \dfrac{1}{x}$

Vertical asymptote: $x = 0$

Slant asymptote: $y = 2x$

Origin symmetry

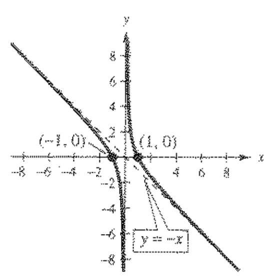


44. $g(x) = \dfrac{1-x^2}{x}$

Intercepts: $(1, 0), (-1, 0)$

Vertical asymptote: $x = 0$

Slant asymptote: $y = -x$

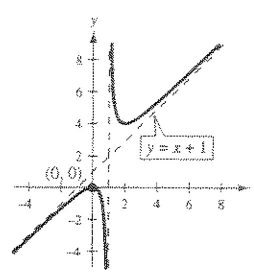


45. $h(x) = \dfrac{x^2}{x-1} = x + 1 + \dfrac{1}{x-1}$

Intercept: $(0, 0)$

Vertical asymptote: $x = 1$

Slant asymptote: $y = x + 1$



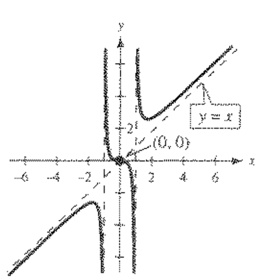
46. $f(x) = \dfrac{x^3}{x^2-1} = x + \dfrac{x}{x^2-1}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = \pm 1$

Slant asymptote: $y = x$

Origin symmetry



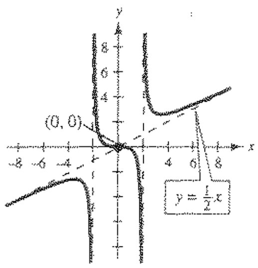
47. $g(x) = \dfrac{x^3}{2x^2 - 8} = \dfrac{1}{2}x + \dfrac{4x}{2x^2 - 8}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = \pm 2$

Slant asymptote: $y = \dfrac{1}{2}x$

Origin symmetry

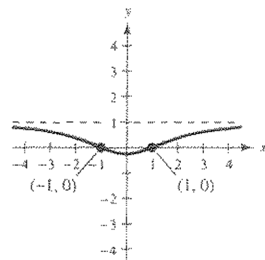


48. $f(x) = \dfrac{x^2 - 1}{x^2 + 4}$

No vertical asymptotes

Horizontal asymptote: $y = 1$

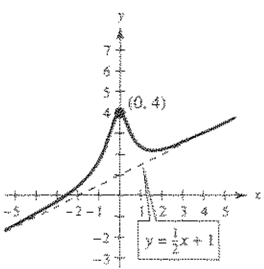
Intercepts: $(\pm 1, 0), \left(0, -\dfrac{1}{4}\right)$



49. $f(x) = \dfrac{x^3 + 2x^2 + 4}{2x^2 + 1} = \dfrac{x}{2} + 1 + \dfrac{3 - \frac{x}{2}}{2x^2 + 1}$

Intercepts: $(-2.594, 0), (0, 4)$

Slant asymptote: $y = \dfrac{x}{2} + 1$

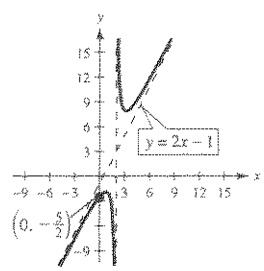


50. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2} = 2x - 1 + \dfrac{3}{x - 2}$

y-intercept: $\left(0, -\dfrac{5}{2}\right)$

Vertical asymptote: $x = 2$

Slant asymptote: $y = 2x - 1$



51. $y = \dfrac{x + 1}{x - 3}$

(a) x-intercept: $(-1, 0)$

(b) $0 = \dfrac{x + 1}{x - 3}$

$0 = x + 1$

$-1 = x$

52. $y = \dfrac{2x}{x - 3}$

(a) x-intercept: $(0, 0)$

(b) $0 = \dfrac{2x}{x - 3}$

$0 = 2x$

$0 = x$

53. $y = \dfrac{1}{x} - x$

(a) x-intercepts: $(\pm 1, 0)$

(b) $0 = \dfrac{1}{x} - x$

$x = \dfrac{1}{x}$

$x^2 = 1$

$x = \pm 1$

54. $y = x - 3 + \frac{2}{x}$

(a) x-intercepts: $(1, 0), (2, 0)$

(b) $0 = x - 3 + \frac{2}{x}$

$0 = x^2 - 3x + 2$

$0 = (x - 1)(x - 2)$

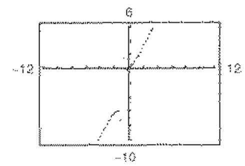
$x = 1, 2$

55. $y = \frac{2x^2 + x}{x + 1} = 2x - 1 + \frac{1}{x + 1}$

Domain: all real numbers except $x = -1$

Vertical asymptote: $x = -1$

Slant asymptote: $y = 2x - 1$

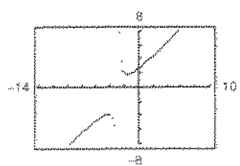


56. $y = \frac{x^2 + 5x + 8}{x + 3} = x + 2 + \frac{2}{x + 3}$

Domain: all $x \neq -3$

Vertical asymptote: $x = -3$

Slant asymptote: $y = x + 2$



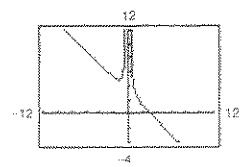
57. $y = \frac{1 + 3x^2 - x^3}{x^2} = \frac{1}{x^2} + 3 - x = -x + 3 + \frac{1}{x^2}$

Domain: all real numbers except 0

or $(-\infty, 0) \cup (0, \infty)$

Vertical asymptote: $x = 0$

Slant asymptote: $y = -x + 3$



58. $y = \frac{12 - 2x - x^2}{2(4 + x)} = -\frac{1}{2}x + 1 + \frac{2}{4 + x}$

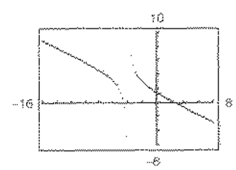
Domain: all real numbers except -4 or $(-\infty, -4) \cup (-4, \infty)$

x-intercepts: $(-4.61, 0), (2.61, 0)$

y-intercept: $\left(0, \frac{3}{2}\right)$

Vertical asymptote: $x = -4$

Slant asymptote: $y = -\frac{1}{2}x + 1$



59. $f(x) = \frac{x^2 - 5x + 4}{x^2 - 4} = \frac{(x - 4)(x - 1)}{(x - 2)(x + 2)}$

Vertical asymptotes: $x = 2, x = -2$

Horizontal asymptote: $y = 1$

No slant asymptotes, no holes

60. $f(x) = \frac{x^2 - 2x - 8}{x^2 - 9} = \frac{(x - 4)(x + 2)}{(x - 3)(x + 3)}$

Vertical asymptotes: $x = 3, x = -3$

Horizontal asymptote: $y = 1$

No slant asymptotes, no holes

61. $f(x) = \dfrac{2x^2 - 5x + 2}{2x^2 - x - 6} = \dfrac{(2x-1)(x-2)}{(2x+3)(x-2)} = \dfrac{2x-1}{2x+3},$

$x \neq 2$

Vertical asymptote: $x = -\dfrac{3}{2}$

Horizontal asymptote: $y = 1$

No slant asymptotes

Hole at $x = 2$, $\left(2, \dfrac{3}{7}\right)$

62. $f(x) = \dfrac{3x^2 - 8x + 4}{2x^2 - 3x - 2} = \dfrac{(3x-2)(x-2)}{(2x+1)(x-2)} = \dfrac{3x-2}{2x+1},$

$x \neq 2$

Vertical asymptote: $x = -\dfrac{1}{2}$

Horizontal asymptote: $y = \dfrac{3}{2}$

No slant asymptotes

Hole at $x = 2$, $\left(2, \dfrac{4}{5}\right)$

63. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

$= \dfrac{(x-1)(x+1)(2x-1)}{(x+1)(x+2)}$

$= \dfrac{(x-1)(2x-1)}{x+2},\ x \neq -1$

Long division gives

$f(x) = \dfrac{2x^2 - 3x + 1}{x + 2} = 2x - 7 + \dfrac{15}{x+2}.$

Vertical asymptote: $x = -2$

No horizontal asymptote

Slant asymptote: $y = 2x - 7$

Hole at $x = -1$, $(-1, 6)$

64. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$

$= \dfrac{(x-2)(x+2)(2x+1)}{(x-2)(x-1)}$

$= \dfrac{(x+2)(2x+1)}{x-1},\ x \neq 2$

Long division gives $f(x) = 2x + 7 + \dfrac{9}{x-1}.$

Vertical asymptote: $x = 1$

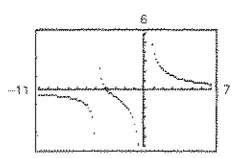
No horizontal asymptote

Slant asymptote: $y = 2x + 7$

Hole at $x = 2$, $(2, 20)$

65. $y = \dfrac{1}{x+5} + \dfrac{4}{x}$

(a)



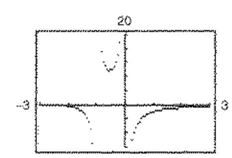
x-intercept: $(-4, 0)$

(b)

$$0 = \frac{1}{x+5} + \frac{4}{x}$$
$$-\frac{4}{x} = \frac{1}{x+5}$$
$$-4(x+5) = x$$
$$-4x - 20 = x$$
$$-5x = 20$$
$$x = -4$$

66. $y = \dfrac{2}{x+1} - \dfrac{3}{x}$

(a)



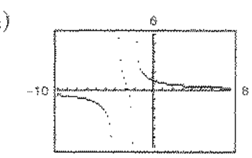
x-intercept: $(-3, 0)$

(b)

$$\frac{2}{x+1} - \frac{3}{x} = 0$$
$$\frac{2}{x+1} = \frac{3}{x}$$
$$2x = 3x + 3$$
$$-3 = x$$

67. $y = \dfrac{1}{x+2} + \dfrac{2}{x+4}$

(a)



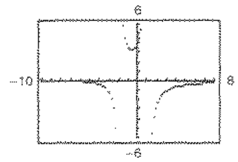
x-intercept: $\left(-\dfrac{8}{3}, 0\right)$

(b)

$$\frac{1}{x+2} + \frac{2}{x+4} = 0$$
$$\frac{1}{x+2} = \frac{-2}{x+4}$$
$$x + 4 = -2x - 4$$
$$3x = -8$$
$$x = -\frac{8}{3}$$

68. $y = \dfrac{2}{x+2} - \dfrac{3}{x-1}$

(a)



x-intercept: $(-8, 0)$

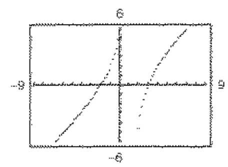
(b) $\dfrac{2}{x+2} = \dfrac{3}{x-1}$

$2x - 2 = 3x + 6$

$-8 = x$

69. $y = x - \dfrac{6}{x-1}$

(a)



x-intercept: $(-2, 0), (3, 0)$

(b) $0 = x - \dfrac{6}{x-1}$

$\dfrac{6}{x-1} = x$

$6 = x(x-1)$

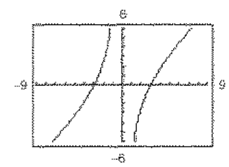
$0 = x^2 - x - 6$

$0 = (x+2)(x-3)$

$x = -2, \quad x = 3$

70. $y = x - \dfrac{9}{x}$

(a)



x-intercepts: $(-3, 0), (3, 0)$

(b) $0 = x - \dfrac{9}{x}$

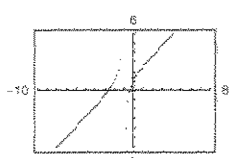
$\dfrac{9}{x} = x$

$9 = x^2$

$\pm 3 = x$

71. $y = x + 2 - \dfrac{1}{x+1}$

(a)



x-intercepts: $(-2.618, 0), (-0.382, 0)$

(b) $x + 2 = \dfrac{1}{x+2}$

$x^2 + 3x + 2 = 1$

$x^2 + 3x + 1 = 0$

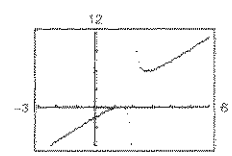
$x = \dfrac{-3 \pm \sqrt{9-4}}{2}$

$= \dfrac{-3}{2} \pm \dfrac{\sqrt{5}}{2}$

$\approx -2.618, -0.382$

72. $y = 2x - 1 + \dfrac{1}{x-2}$

(a)



x-intercepts: $(1, 0), \left(\dfrac{3}{2}, 0\right)$

(b) $2x - 1 + \dfrac{1}{x-2} = 0$

$\dfrac{1}{x-2} = 1 - 2x$

$1 = -2x^2 + 5x - 2$

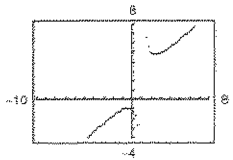
$2x^2 - 5x + 3 = 0$

$(x-1)(2x-3) = 0$

$x = 1, \dfrac{3}{2}$

73. $y = x + 1 + \dfrac{2}{x - 1}$

(a)



No x-intercepts

(b) $x + 1 + \dfrac{2}{x - 1} = 0$

$$\frac{2}{x-1} = -x - 1$$

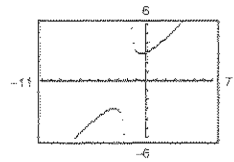
$$2 = -x^2 + 1$$

$x^2 + 1 = 0$

No real zeros

74. $y = x + 2 + \dfrac{2}{x + 2}$

(a)



No x-intercept

(b) $x + 2 + \dfrac{2}{x + 2} = 0$

$$\frac{2}{x+2} = -x - 2$$

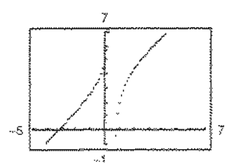
$$2 = -x^2 - 4x - 4$$

$x^2 + 4x + 6 = 0$

Because $b^2 - 4ac = 16 - 24 < 0$, there are no real zeros.

75. $y = x + 3 - \dfrac{2}{2x - 1}$

(a)



x-intercepts: $(0.766, 0)$, $(-3.266, 0)$

(b) $x + 3 - \dfrac{2}{2x - 1} = 0$

$$x + 3 = \frac{2}{2x - 1}$$

$$2x^2 + 5x - 3 = 2$$

$$2x^2 + 5x - 5 = 0$$

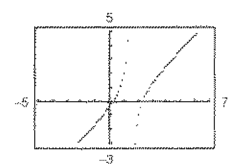
$$x = \frac{-5 \pm \sqrt{25 - 4(2)(-5)}}{4}$$

$$= \frac{-5 \pm \sqrt{65}}{4}$$

$$\approx 0.766, -3.266$$

76. $y = x - 1 - \dfrac{2}{2x - 3}$

(a)



x-intercepts: $(0.219, 0)$, $(2.281, 0)$

(b) $x - 1 - \dfrac{2}{2x - 3} = 0$

$$x - 1 = \frac{2}{2x - 3}$$

$$2x^2 - 5x + 3 = 2$$

$$2x^2 - 5x + 1 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 8}}{4}$$

$$= \frac{5 \pm \sqrt{17}}{4} \approx 0.219, 2.281$$

77. (a) $0.25(50) + 0.75(x) = C(50 + x)$

$$\frac{12.5 + 0.75x}{50 + x} = C$$

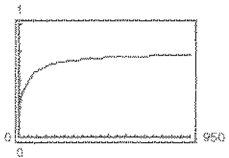
$$\frac{50 + 3x}{200 + 4x} = C$$

$$C = \frac{3x + 50}{4(x + 50)}$$

(b) Domain: $x \geq 0$ and $x \leq 1000 - 50 = 950$

Thus, $0 \leq x \leq 950$.

(c)



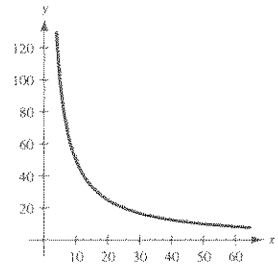
As the tank fills, the rate that the concentration is increasing slows down. It approaches the horizontal asymptote $C = \frac{3}{4} = 0.75$. When the tank is full ($x = 950$), the concentration is $C = 0.725$.

78. (a) Area $= xy = 500$

$$y = \frac{500}{x}$$

(b) Domain: $x > 0$

(c)



For $x = 30$, $y = \frac{500}{30} = 16\frac{2}{3}$ meters.

79. (a) $A = xy$ and

$$(x - 2)(y - 4) = 30$$

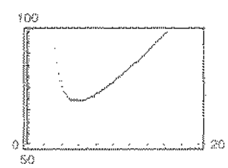
$$y - 4 = \frac{30}{x - 2}$$

$$y = 4 + \frac{30}{x - 2} = \frac{4x + 22}{x - 2}$$

Thus, $A = xy = x\left(\frac{4x + 22}{x - 2}\right) = \frac{2x(2x + 11)}{x - 2}$.

(b) Domain: Since the margins on the left and right are each 1 inch, $x > 2$, or $(2, \infty)$.

(c)



The area is minimum when $x \approx 5.87$ in. and $y \approx 11.75$ in.

80. (a) The line passes through the points $(a, 0)$ and $(3, 2)$ and has a slope of

$$m = \frac{2 - 0}{3 - a} = \frac{2}{3 - a}.$$

$$y - 0 = \frac{2}{3 - a}(x - a) \text{ by the point-slope form}$$

$$y = \frac{2(x - a)}{3 - a} = \frac{-2(a - x)}{-1(a - 3)}$$

$$= \frac{2(a - x)}{a - 3},\ 0 \leq x \leq a$$

(b) The area of a triangle is $A = \frac{1}{2}bh$.

$b = a$

$h = y$ when $x = 0$, so $h = \dfrac{2(a - 0)}{a - 3} = \dfrac{2a}{a - 3}$.

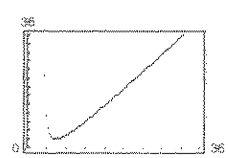
$$A = \frac{1}{2}a\left(\frac{2a}{a - 3}\right) = \frac{a^2}{a - 3}$$

(c) $A = \dfrac{a^2}{a - 3} = a + 3 + \dfrac{9}{a - 3}$

Vertical asymptote: $a = 3$

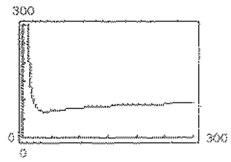
Slant asymptote: $A = a + 3$

A is a minimum when $a = 6$ and $A = 12$.



81. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x+30}\right),\ 1 \le x$

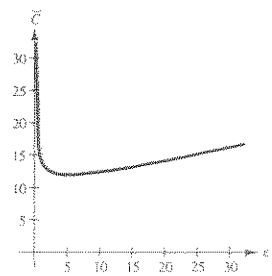
The minimum occurs when $x \approx 40.4 \approx 40$.



82. $\overline{C} = \dfrac{C}{x} = \dfrac{0.2x^2 + 10x + 5}{x},\ x > 0$

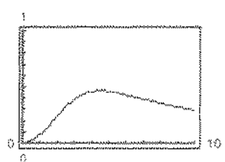
The minimum average cost occurs when $x = 5$.

x	0.5	1	2	3	4	5	6	7
$\overline{C}$	20.1	15.2	12.9	12.3	12.05	12	≈ 12.0	12.1



83. $C = \dfrac{3t^2 + t}{t^3 + 50},\ 0 \le t$

(a) The horizontal asymptote is the t-axis, or $C = 0$. This indicates that the chemical eventually dissipates.

(b) 

The maximum occurs when $t \approx 4.5$.

(c) Graph C together with $y = 0.345$. The graphs intersect at $t \approx 2.65$ and $t \approx 8.32$. $C < 0.345$ when $0 \le t < 2.65$ hours and when $t > 8.32$ hours.

84. (a) Rate $\times$ Time $=$ Distance or $\dfrac{\text{Distance}}{\text{Rate}} = \text{Time}$

$$\frac{100}{x} + \frac{100}{y} = \frac{200}{50} = 4$$

$$\frac{25}{x} + \frac{25}{y} = 1$$

$$25y + 25x = xy$$

$$25x = xy - 25y$$

$$25x = y(x - 25)$$

$$y = \frac{25x}{x - 25}$$

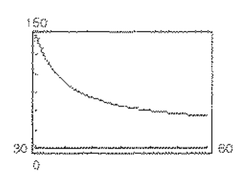
(b) Vertical asymptote: $x = 25$

Horizontal asymptote: $y = 25$

(c)

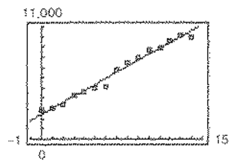
x	30	35	40	45	50	55	60
y	150	87.5	66.7	56.3	50	45.8	42.9

The results in the table are unexpected. You would expect the average speed for the round trip to be the average of the average speeds for the two parts of the trip.

(d) 

(e) No, it is not possible to average 20 miles per hour in one direction and still average 50 miles per hour on the round trip. At 20 miles per hour you would use more time in one direction than is required for the round trip at an average speed of 50 miles per hour.

85. (a) $y_1 = 583.8t + 2414$ ($t = 0$ corresponds to 1990)

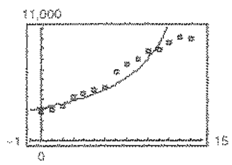


(b) Using the data $\left(t = \frac{1}{A}\right)$, we obtain:

$y_2 = -0.00001855t + 0.0003150$

$y_3 = \dfrac{1}{-0.00001855t + 0.000315} = \dfrac{1}{A}$

(c)



86. (a) Domain: $t \geq 0$

(b) At $t = 0$, $P = 10$.

(c) $P(25) \approx 22$ elk

$P(50) \approx 24$ elk

$P(100) \approx 25$ elk

(d) Yes, the horizontal aymptote $y = \dfrac{2.7}{0.1} \approx 27$ is the limit.

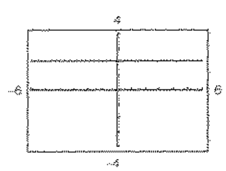
87. False, you will have to lift your pencil to cross the vertical asymptote.

88. False. $f(x) = \dfrac{x}{x^3 + 1}$ crosses its horizontal asymptote $y = 0$ at $x = 0$.

89. $h(x) = \dfrac{6 - 2x}{3 - x} = \dfrac{2(3 - x)}{3 - x} = 2, \quad x \neq 3$

Since $h(x)$ is not reduced and $(3 - x)$ is a factor of both the numerator and the denominator, $x = 3$ is not a horizontal asymptote.

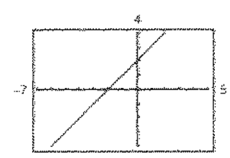
There is a hole in the graph at $x = 3$.



90. $g(x) = \dfrac{x^2 + x - 2}{x - 1}$

$= \dfrac{(x + 2)(x - 1)}{x - 1} = x + 2, x \neq 1$

Since $g(x)$ is not reduced $(x - 1)$ is a factor of both the numerator and the denominator, $x = 1$ is not a horizontal asymptote.



There is a hole at $x = 1$.

91. $y = x + 1 + \dfrac{a}{x + 2}$ has a slant asymptote $y = x + 1$ and a vertical asymptote $x = -2$.

$0 = 2 + 1 + \dfrac{a}{2 + 2}$

$0 = 3 + \dfrac{a}{4}$

$\dfrac{a}{4} = -3$

$a = -12$

Hence, $y = x + 1 - \dfrac{12}{x + 2} = \dfrac{x^2 + 3x - 10}{x + 2}$.

92. $y = x - 2 + \dfrac{a}{x + 4}$ has slant asymptote $y = x - 2$ and vertical asymptote at $x = -4$. We determine a so that y has a zero at $x = 3$:

$0 = 3 - 2 + \dfrac{a}{3 + 4} = 1 + \dfrac{a}{7} \Rightarrow a = -7$

Hence, $y = x - 2 + \dfrac{-7}{x + 4} = \dfrac{x^2 + 2x - 15}{x + 4}$.

93. $\left(\frac{x}{8}\right)^{-3} = \left(\frac{8}{x}\right)^3 = \frac{512}{x^3}$

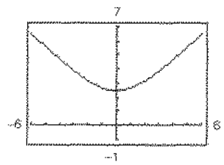
94. $(4x^2)^{-2} = \frac{1}{(4x^2)^2} = \frac{1}{16x^4}$

95. $\frac{3^{7/6}}{3^{1/6}} = 3^{6/6} = 3$

96. $\frac{x^{-2} \cdot x^{1/2}}{x^{-1} \cdot x^{5/2}} = \frac{x \cdot x^{1/2}}{x^2 \cdot x^{5/2}} = \frac{x^{3/2}}{x^{9/2}}$

$= \frac{1}{x^3}$

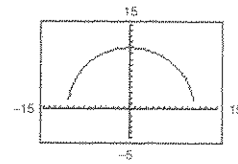
97.



Domain: all x

Range: $y \geq \sqrt{6}$

98.

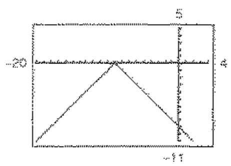


Semicircle

Domain: $-11 \leq x \leq 11$

Range: $0 \leq y \leq 11$

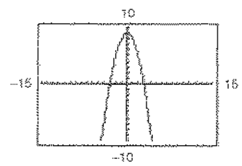
99.



Domain: all x

Range: $y \leq 0$

100.



Parabola

Domain: all x

Range: $y \leq 9$

101. Answers will vary. (Make a Decision)

Section 2.8 Quadratic Models

You should know how to

- Construct and classify scatter plots.
- Fit a quadratic model to data.
- Choose an appropriate model given a set of data.

Vocabulary Check

1. linear

2. quadratic

1. A quadratic model is better.

2. Linear

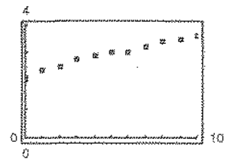
3. A linear model is better.

4. Neither

5. Neither linear nor quadratic

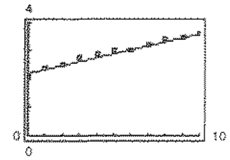
6. Quadratic

7. (a)



(b) Linear model is better.

(d)



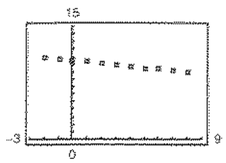
(c) $y = 0.14x + 2.2$, linear

$[y = -0.00478x^2 + 0.1887x + 2.1692$, quadratic$]$

(e)

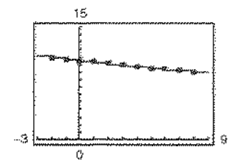
x	0	1	2	3	4	5	6	7	8	9	10
y	2.1	2.4	2.5	2.8	2.9	3.0	3.0	3.2	3.4	3.5	3.6
Model	2.2	2.4	2.5	2.7	2.8	2.9	3.1	3.2	3.4	3.5	3.6

8. (a)



(b) Quadratic model is better. Answers will vary.

(d)



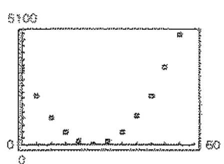
(c) $y = -0.19x + 10.5$, linear

$y = 0.006x^2 - 0.23x + 10.5$, quadratic

(e)

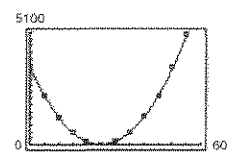
x	-2	-1	0	1	2	3	4	5	6	7	8
y	11	10.7	10.4	10.3	10.1	9.9	9.6	9.4	9.4	9.2	9.0
Model	11	10.7	10.4	10.3	10.1	9.9	9.7	9.5	9.3	9.2	9.0

9. (a)



(b) Quadratic model is better.

(d)

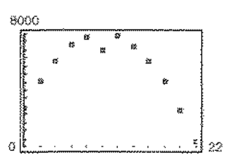


(c) $y = 5.55x^2 - 277.5x + 3478$

(e)

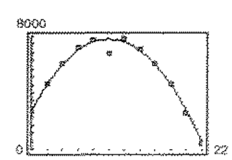
x	0	5	10	15	20	25	30	35	40	45	50	55
y	3480	2235	1250	565	150	12	145	575	1275	2225	3500	5010
Model	3478	2229	1258	564	148	9	148	564	1258	2229	3478	5004

10. (a)



(b) Quadratic

(d)



(c) $y = -17.793x^2 + 354.797x + 6162.9$

—CONTINUED—

10. —CONTINUED—

(e)

x	0	2	4	6	8	10	12	14	16	18	20	22
y	6140	6815	7335	7710	7915	7590	7975	7700	7325	6820	6125	5325
Model	6163	6801	7297	7651	7863	7932	7858	7643	7285	6784	6142	5357

(Answers will vary.)

11. (a) $y = 2.48x + 1.1$, linear

$y = 0.071x^2 + 1.69x + 2.7$, quadratic

(b) 0.98995 for linear model

0.99519 for quadratic model

(c) Quadratic fits better.

12. (a) $y = 2.10x$, linear model

$y = 0.006x^2 + 2.04x$, quadratic model

(b) 0.99978 for the linear model

0.99984 for the quadratic model

(c) The quadratic model is slightly better.

13. (a) $y = -0.89x + 5.3$, linear

$y = 0.001x^2 - 0.90x + 5.3$, quadratic

(b) 0.99982 for the linear model

0.99987 for the quadratic model

(c) The quadratic model is slightly better.

14. (a) $y = -8.6x + 612$

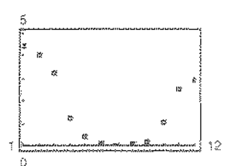
$y = 0.08x^2 - 9.0x + 595$

(b) 0.98235, linear

0.99653, quadratic

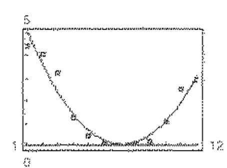
(c) Quadratic is better.

15. (a)



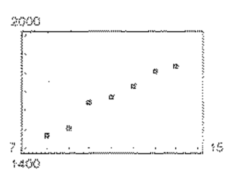
(b) $P = 0.1322t^2 - 1.901t + 6.87$

(c)



(d) The model's minimum is $H \approx 0.03$ at $t = 7.2$. This corresponds to July.

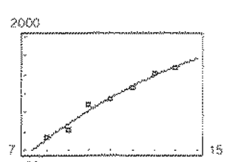
16. (a)



(b) $S = -3.07t^2 + 131.9t + 597$

($t = 8 \leftrightarrow 1998$)

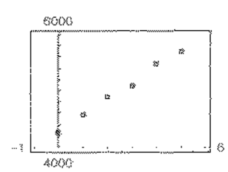
(c)



(d) Using a graphics utility, $S > 2000$ when $t \approx 19.4$, or 2009.

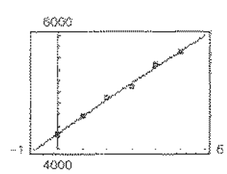
(e) No, because the coefficient of t^2 is negative. The graph turns downward as time increases. Answers will vary.

17. (a)



(b) $y = -2.630t^2 + 301.74t + 4270.2$

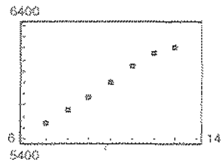
(c)



(d) According to the model, $y > 10{,}000$ when $t \approx 24$, or 2024.

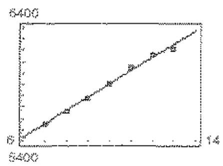
(e) Answers will vary.

18. (a)



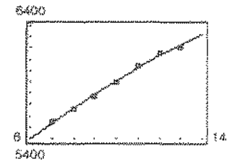
(b) $S = 117.1t + 4727$, linear model (0.98921)

(c)



(d) $S_2 = -3.26t^2 + 182.3t + 4413$ (0.99151)

(e)

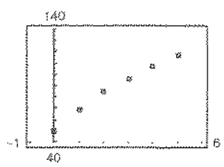


(f) Answers will vary.

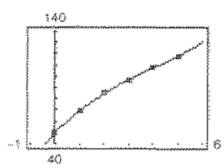
(g) $S_1 > 7000$ for $t > 19.4$, or 2009.

S_2 is never greater than 7000.

19. (a)

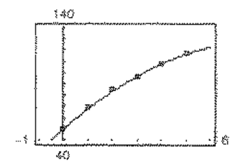


(b)



(c) $y = -1.1357t^2 + 18.999t + 50.32$, quadratic model, (0.99859)

(d)

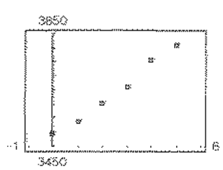


(e) The cubic model is a better fit.

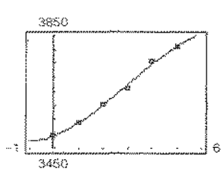
(f)

Year	2006 ($t = 6$)	2007 ($t = 7$)	2008 ($t = 8$)
A^*	127.76	140.15	154.29
Cubic	129.91	145.13	164.96
Quadratic	123.40	127.64	129.60

20. (a)



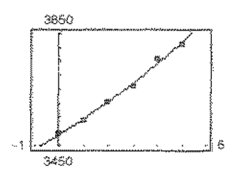
(b)



Answers will vary.

(c) $y = 2.36t^2 + 53.7t + 3489$ quadratic model, (0.99474)

(d)



Answers will vary.

(e) Cubic model is better.

(f)

Year	2006 ($t = 6$)	2007 ($t = 7$)	2008 ($t = 8$)
A^*	3890	3949	4059
Cubic	3858	3878	3862
Quadratic	3896	3981	4070

21. True

22. True

23. The model is above all data points.

24. (a) $(f \circ g)(x) = f(x^2 + 3) = 2(x^2 + 3) - 1 = 2x^2 + 5$

(b) $(g \circ f)(x) = g(2x - 1) = (2x - 1)^2 + 3 = 4x^2 - 4x + 4$

25. (a) $f(g(x)) = f(2x^2 - 1) = 5(2x^2 - 1) + 8 = 10x^2 + 3$

(b) $g(f(x)) = g(5x + 8) = 2(5x + 8)^2 - 1 = 50x^2 + 160x + 127$

26. (a) $(f \circ g)(x) = f\left(\sqrt[3]{x+1}\right) = x + 1 - 1 = x$

(b) $(g \circ f)(x) = g(x^3 - 1) = \sqrt[3]{x^3 - 1 + 1} = x$

27. (a) $f(g(x)) = f(x^3 - 5) = \sqrt[3]{x^3 - 5 + 5} = x$

(b) $g(f(x)) = g\left(\sqrt[3]{x+5}\right) = \left[\sqrt[3]{x+5}\right]^3 - 5 = x$

28. f is one-to-one.

$$y = 2x + 5$$
$$x = 2y + 5$$
$$2y = x - 5$$
$$y = \frac{(x-5)}{2} \implies f^{-1}(x) = \frac{x-5}{2}$$

29. f is one-to-one.

$$y = \frac{x-4}{5}$$
$$x = \frac{y-4}{5}$$
$$5x + 4 = y \implies f^{-1}(x) = 5x + 4$$

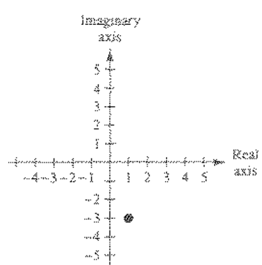
30. f is one-to-one on $[0, \infty)$.

$$y = x^2 + 5, \quad x \geq 0$$
$$x = y^2 + 5, \quad y \geq 0$$
$$y^2 = x - 5$$
$$y = \sqrt{x-5} \implies f^{-1}(x) = \sqrt{x-5}, \quad x \geq 5$$

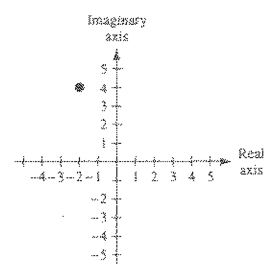
31. f is one-to-one.

$$y = 2x^2 - 3, \quad x \geq 0$$
$$x = 2y^2 - 3, \quad y \geq 0$$
$$y^2 = \frac{(x+3)}{2}$$
$$y = \sqrt{\frac{x+3}{2}} \implies f^{-1}(x) = \sqrt{\frac{x+3}{2}}$$
$$= \frac{\sqrt{2x+6}}{2}, \quad x \geq -3$$

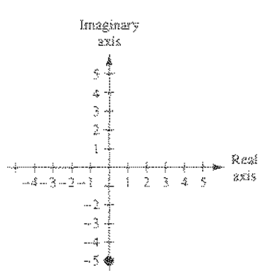
32.



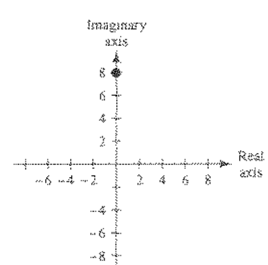
33.



34.

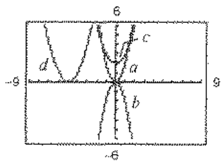


35.



Review Exercises for Chapter 2

1.



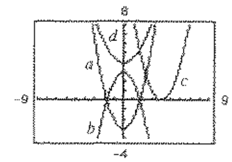
(a) $y = 2x^2$ is a vertical stretch.

(b) $y = -2x^2$ is a vertical stretch and reflection in the x-axis.

(c) $y = x^2 + 2$ is a vertical shift two units upward.

(d) $y = (x + 5)^2$ is a horizontal shift five units to the left.

2.



(a) Vertical shift three units downward

(b) Reflection in the x-axis and vertical shift three units upward

(c) Horizontal shift four units to the right

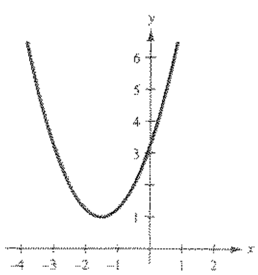
(d) Vertical shrink followed by vertical shift four units upward

3. $f(x) = \left(x + \frac{3}{2}\right)^2 + 1$

Vertex: $\left(-\frac{3}{2}, 1\right)$

y-intercept: $\left(0, \frac{13}{4}\right)$

No x-intercepts

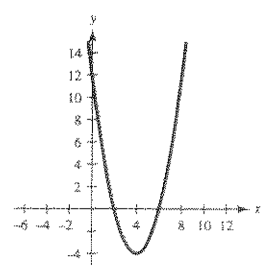


4. $f(x) = (x - 4)^2 - 4$

Vertex: $(4, -4)$

y-intercept: $(0, 12)$

x-intercepts: $(2, 0)$, $(6, 0)$



5. $f(x) = \dfrac{1}{3}(x^2 + 5x - 4)$

$$= \frac{1}{3}\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 4\right)$$

$$= \frac{1}{3}\left[\left(x + \frac{5}{2}\right)^2 - \frac{41}{4}\right]$$

$$= \frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12}$$

Vertex: $\left(-\dfrac{5}{2}, -\dfrac{41}{12}\right)$

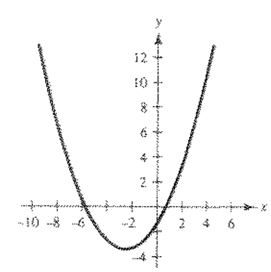
y-intercept: $\left(0, -\dfrac{4}{3}\right)$

x-intercepts: $0 = \dfrac{1}{3}(x^2 + 5x - 4)$

$$0 = x^2 + 5x - 4$$

$$x = \frac{-5 \pm \sqrt{41}}{2} \quad \text{Use the Quadratic Formula.}$$

$$\left(\frac{-5 \pm \sqrt{41}}{2}, 0\right)$$



6. $f(x) = 3x^2 - 12x + 11$

$$= 3\left(x^2 - 4x + 4 - 4 + \frac{11}{3}\right)$$

$$= 3\left[(x-2)^2 - \frac{1}{3}\right]$$

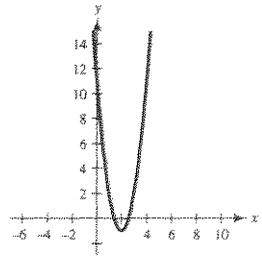
$$= 3(x-2)^2 - 1$$

Vertex: $(2, -1)$

y-intercept: $(0, 11)$

x-intercepts: $x = \dfrac{12 \pm \sqrt{12}}{6} = 2 \pm \dfrac{1}{3}\sqrt{3}$

$\left(2 + \frac{1}{3}\sqrt{3}, 0\right), \left(2 - \frac{1}{3}\sqrt{3}, 0\right)$



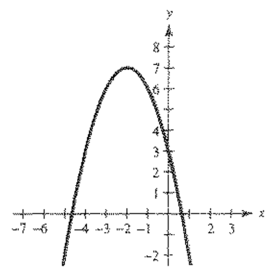
7. $f(x) = 3 - x^2 - 4x$

$$= 3 - (x^2 + 4x + 4) + 4$$

$$= 7 - (x+2)^2$$

Vertex: $(-2, 7)$

Intercepts: $(0, 3), \left(-2 \pm \sqrt{7}, 0\right)$



8. $f(x) = 3\left(x^2 + \frac{23}{3}x + \frac{529}{36}\right) + 30 - \frac{529}{12}$

$$= 3\left(x + \tfrac{23}{6}\right)^2 - \tfrac{169}{12}$$

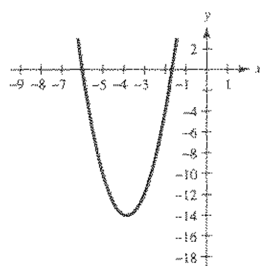
Vertex: $\left(-\frac{23}{6}, -\frac{169}{12}\right)$

$$3\left(x + \tfrac{23}{6}\right)^2 = \tfrac{169}{12}$$

$$x + \tfrac{23}{6} = \pm\sqrt{\tfrac{169}{36}} = \pm\tfrac{13}{6}$$

$$x = -6, -\tfrac{5}{3}$$

Intercepts: $(-6, 0), \left(-\frac{5}{3}, 0\right), (0, 30)$



9. Vertex: $(1, -4) \implies f(x) = a(x-1)^2 - 4$

Point: $(2, -3) \implies -3 = a(2-1)^2 - 4$

$$1 = a$$

Thus, $f(x) = (x-1)^2 - 4$.

10. Vertex: $(2, 3) \implies y = a(x-2)^2 + 3$

Point: $(0, 2) \implies 2 = a(0-2)^2 + 3$

$$= 4a + 3 \implies a = -\tfrac{1}{4}$$

$y = -\frac{1}{4}(x-2)^2 + 3$

11. Vertex: $(-2, -2) \implies f(x) = a(x+2)^2 - 2$

Point: $(-1, 0) \implies 0 = a(-1+2)^2 - 2$

$$a = 2$$

Thus, $f(x) = 2(x+2)^2 - 2$.

12. Vertex: $\left(-\frac{1}{4}, \frac{3}{2}\right) \implies y = a\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}$

Point: $(-2, 0) \implies 0 = a\left(-2 + \frac{1}{4}\right)^2 + \frac{3}{2}$

$$= \tfrac{49}{16}a + \tfrac{3}{2} \implies a = -\tfrac{24}{49}$$

$y = -\frac{24}{49}\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}$

13. (a) $A = xy = x\left(\frac{8 - x}{2}\right)$, since $x + 2y - 8 = 0 \implies y = \frac{8 - x}{2}$.

Since the figure is in the first quadrant and x and y must be positive, the domain of

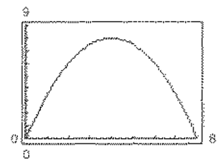
$A = x\left(\frac{8 - x}{2}\right)$ is $0 < x < 8$.

(b)

x	y	Area
1	$4 - \frac{1}{2}(1)$	$(1)\left[4 - \frac{1}{2}(1)\right] = \frac{7}{2}$
2	$4 - \frac{1}{2}(2)$	$(2)\left[4 - \frac{1}{2}(2)\right] = 6$
3	$4 - \frac{1}{2}(3)$	$(3)\left[4 - \frac{1}{2}(3)\right] = \frac{15}{2}$
4	$4 - \frac{1}{2}(4)$	$(4)\left[4 - \frac{1}{2}(4)\right] = 8$
5	$4 - \frac{1}{2}(5)$	$(5)\left[4 - \frac{1}{2}(5)\right] = \frac{15}{2}$
6	$4 - \frac{1}{2}(6)$	$(6)\left[4 - \frac{1}{2}(6)\right] = 6$

The dimensions that will produce a maximum area seem to be $x = 4$ and $y = 2$.

(c)



The maximum area of 8 occurs at the vertex when $x = 4$ and $y = \frac{8 - 4}{2} = 2$.

(d)
$$\begin{aligned} A &= x\left(\frac{8 - x}{2}\right) \\ &= \frac{1}{2}(8x - x^2) \\ &= -\frac{1}{2}(x^2 - 8x) \\ &= -\frac{1}{2}(x^2 - 8x + 16 - 16) \\ &= -\frac{1}{2}[(x - 4)^2 - 16] \\ &= -\frac{1}{2}(x - 4)^2 + 8 \end{aligned}$$

The maximum area of 8 occurs when $x = 4$ and $y = \frac{8 - 4}{2} = 2$.

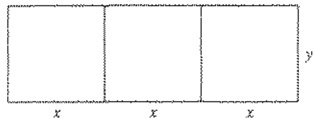
(e) The answers are the same.

14. $C = 10{,}000 - 110x + 0.45x^2$

x	C
100	3500
120	3280
124	3279.2
130	3305

x	C
121	3278.5
122	3277.8
122.5	3277.8
123	3278.1

The minimum is 122 units.

15. 

$6x + 4y = 1500$ Total amount of fencing

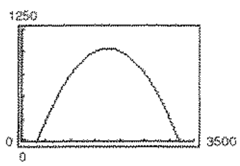
$A = 3xy$ Area enclosed

Because $y = \frac{1}{4}(1500 - 6x)$,

$$A = 3x\left(\frac{1}{4}\right)(1500 - 6x)$$

$$= -\frac{9}{2}x^2 + 1125x.$$

The vertex is at $x = \frac{-b}{2a} = \frac{-1125}{2(-9/2)} = 125$. Thus $x = 125$ feet, $y = \frac{1}{4}(1500 - 6(125)) = 187.5$ and the dimensions are 375 feet by 187.5 feet.

16. (a) 

$P = 1.75x - (0.0005x^2 + 500)$

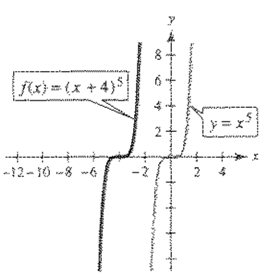
(b) The maximum is $(1750, 1031.25)$.

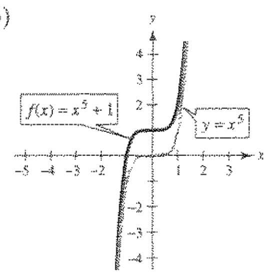
To maximize profit, sell 1750 songs.

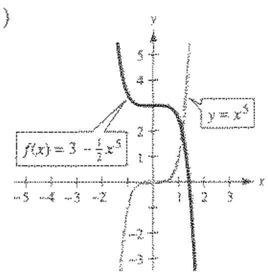
(c) $P = -0.0005x^2 + 1.75x - 500$ opens downward. The vertex is at

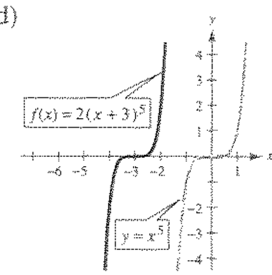
$$x = \frac{-b}{2a} = \frac{-1.75}{-0.001} = 1750.$$

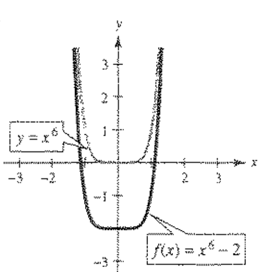
(d) The maximum profit is $P(1750) = \$1031.25$.

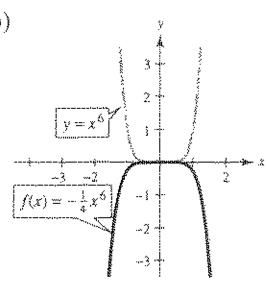
17. (a) 

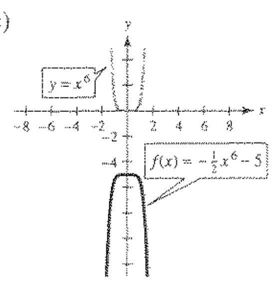
(b) 

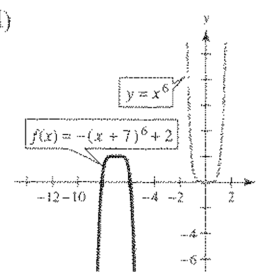
(c) 

(d) 

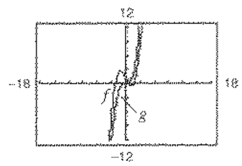
18. (a) 

(b) 

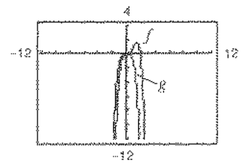
(c) 

(d) 

19. $f(x) = \frac{1}{2}x^3 - 2x + 1$; $g(x) = \frac{1}{2}x^3$



20. $f(x) = -x^4 + 2x^3$; $g(x) = -x^4$



21. $f(x) = -x^2 + 6x + 9$

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

22. $f(x) = \frac{1}{2}x^3 + 2x$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

23. $f(x) = \frac{3}{4}(x^4 + 3x^2 + 2)$

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

24. $h(x) = -x^5 - 7x^2 + 10x$

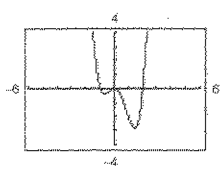
The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

25. (a) $x^4 - x^3 - 2x^2 = x^2(x^2 - x - 2)$

$= x^2(x - 2)(x + 1) = 0$

Zeros: $x = -1, 0, 2$

(b)



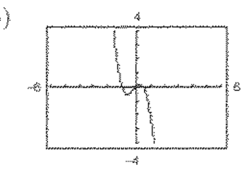
(c) Zeros: $x = -1, 0, 2$; the same

26. (a) $-2x^3 - x^2 + x = -x(2x^2 + x - 1)$

$= -x(2x - 1)(x + 1) = 0$

Zeros: $x = -1, 0, \frac{1}{2}$

(b)

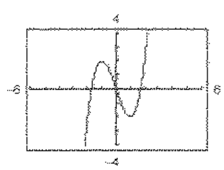


(c) Zeros: $-1, 0, 0.5$; the same

27. (a) $t^3 - 3t = t(t^2 - 3) = t(t + \sqrt{3})(t - \sqrt{3}) = 0$

Zeros: $t = 0, \pm\sqrt{3}$

(b)



(c) Zeros: $t = 0, \pm 1.732$, the same

28. (a) $-(x + 6)^3 - 8 = 0$

$x^3 + 18x^2 + 108x + 224 = 0$

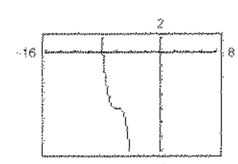
$(x + 8)(x^2 + 10x + 28)$

For the quadratic, $x = \dfrac{-10 \pm \sqrt{100 - 112}}{2}$

$= -5 \pm \sqrt{3}i.$

Zeros: $-8, -5 \pm \sqrt{3}i$

(b)

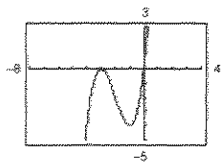


(c) Real zero: $x = -8$

29. (a) $x(x + 3)^2 = 0$

Zeros: $x = 0, -3$

(b)

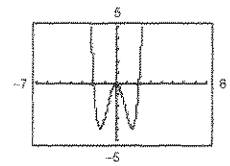


(c) Zeros: $x = -3, 0$, the same

30. (a)
$$t^4 - 4t^2 = 0$$
$$t^2(t^2 - 4) = 0$$
$$t^2(t + 2)(t - 2) = 0$$

Zeros: $t = 0, \pm 2$

(b)



(c) Zeros: $t = 0, \pm 2$, the same

31. $f(x) = (x + 2)(x - 1)^2(x - 5)$

$= x^4 - 5x^3 - 3x^2 + 17x - 10$

32. $f(x) = (x + 3)x(x - 1)(x - 4)$

$= x^4 - 2x^3 - 11x^2 + 12x$

33. $f(x) = (x - 3)(x - 2 + \sqrt{3})(x - 2 - \sqrt{3})$

$= x^3 - 7x^2 + 13x - 3$

34. $f(x) = (x + 7)(x - 4 + \sqrt{6})(x - 4 - \sqrt{6})$

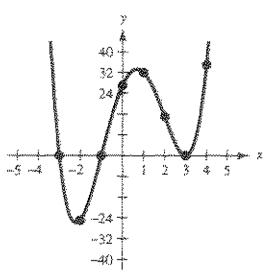
$= x^3 - x^2 - 46x + 70$

35. (a) Degree is even and leading coefficient is $1 > 0$. Rises to the left and rises to the right.

(b) $x^4 - 2x^3 - 12x^2 + 18x + 27 = (x - 3)^2(x + 1)(x + 3)$

Zeros: $\pm 3, -1$

(c) and (d)

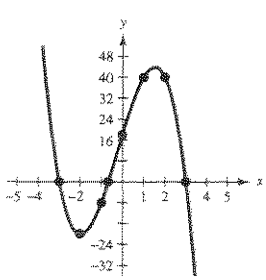


36. (a) Degree is odd and leading coefficient is $-3 < 0$. Rises to the left and falls to the right.

(b) $-3x^3 - 2x^2 + 27x + 18 = -(x - 3)(x + 3)(3x + 2)$

Zeros: $x = \pm 3, -\frac{2}{3}$

(c) and (d)



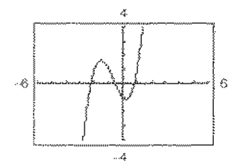
37. $f(x) = x^3 + 2x^2 - x - 1$

(a) $f(-3) < 0,\ f(-2) > 0 \implies$ zero in $[-3, -2]$

$f(-1) > 0,\ f(0) < 0 \implies$ zero in $[-1, 0]$

$f(0) < 0,\ f(1) > 0 \implies$ zero in $[0, 1]$

(b) Zeros: $-2.247, -0.555, 0.802$



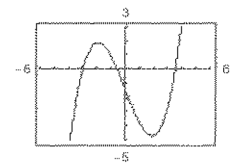
38. (a) $f(x) = 0.24x^3 - 2.6x - 1.4$

$f(-3) < 0, f(-2) > 0 \implies$ zero in $[-3, -2]$

$f(-1) > 0, f(0) < 0 \implies$ zero in $[-1, 0]$

$f(3) < 0, f(4) > 0 \implies$ zero in $[3, 4]$

(b) Zeros: $-2.979, -0.554, 3.533$

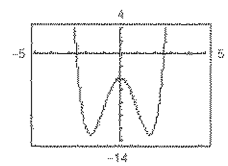


39. $f(x) = x^4 - 6x^2 - 4$

(a) $f(-3) > 0,\ f(-2) < 0 \implies$ zero in $[-3, -2]$

$f(2) < 0,\ f(3) > 0 \implies$ zero in $[2, 3]$

(b) Zeros: ± 2.570

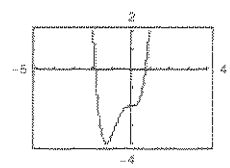


40. $f(x) = 2x^4 + \frac{7}{2}x^3 - 2$

(a) $f(-2) > 0, f(-1) < 0 \implies$ zero in $[-2, -1]$

$f(0) < 0, f(1) > 0 \implies$ zero in $[0, 1]$

(b) Zeros: $-1.897, 0.738$



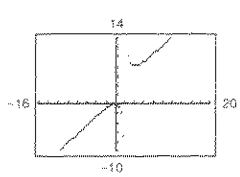
41. $y_1 = \dfrac{x^2}{x - 2}$

$$y_2 = x + 2 + \frac{4}{x - 2}$$

$$= \frac{(x + 2)(x - 2)}{x - 2} + \frac{4}{x - 2}$$

$$= \frac{x^2 - 4}{x - 2} + \frac{4}{x - 2}$$

$$= \frac{x^2}{x - 2} = y_1$$

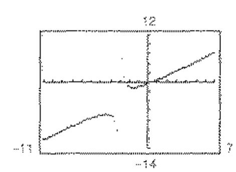


42. $y_1 = \dfrac{x^2 + 2x - 1}{x + 3},\ y_2 = x - 1 + \dfrac{2}{x + 3}$

$$y_2 = x - 1 + \frac{2}{x + 3}$$

$$= \frac{(x - 1)(x + 3) + 2}{x + 3}$$

$$= \frac{x^2 + 2x - 1}{x + 3} = y_1$$



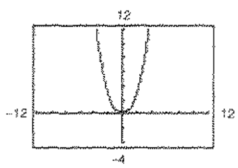
43. $y_1 = \dfrac{x^4 + 1}{x^2 + 2}$

$$y_2 = x^2 - 2 + \frac{5}{x^2 + 2}$$

$$= \frac{x^2(x^2 + 2)}{x^2 + 2} - \frac{2(x^2 + 2)}{x^2 + 2} + \frac{5}{x^2 + 2}$$

$$= \frac{x^4 + 2x^2 - 2x^2 - 4 + 5}{x^2 + 2}$$

$$= \frac{x^4 + 1}{x^2 + 2} = y_1$$

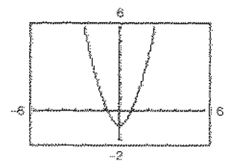


44. $y_1 = \dfrac{x^4 + x^2 - 1}{x^2 + 1}$

$$y_2 = x^2 - \frac{1}{x^2 + 1}$$

$$= \frac{x^2(x^2 + 1) - 1}{x^2 + 1}$$

$$= \frac{x^4 + x^2 - 1}{x^2 + 1} = y_1$$



45.

$$\begin{array}{r}
8x + 5 \\
3x - 2\,\overline{)\,24x^2 - x - 8} \\
\underline{24x^2 - 16x} \\
15x - 8 \\
\underline{15x - 10} \\
2
\end{array}$$

Thus, $\dfrac{24x^2 - x - 8}{3x - 2} = 8x + 5 + \dfrac{2}{3x - 2}$.

46.

$$\begin{array}{r}
\frac{4}{3}x + \frac{8}{9} \\
3x - 2\,\overline{)\,4x^2 + 0x + 7} \\
\underline{4x^2 - \frac{8}{3}x} \\
\frac{8}{3}x + 7 \\
\underline{\frac{8}{3}x - \frac{16}{9}} \\
\frac{79}{9}
\end{array}$$

$$\frac{4x^2 + 7}{3x - 2} = \frac{4}{3}x + \frac{8}{9} + \frac{\frac{79}{9}}{3x - 2} = \frac{4}{3}x + \frac{8}{9} + \frac{79}{27x - 18}$$

47.

$$\begin{array}{r}
x^2 - 2 \\
x^2 - 1\,\overline{)\,x^4 - 3x^2 + 2} \\
\underline{x^4 - x^2} \\
-2x^2 + 2 \\
\underline{-2x^2 + 2} \\
0
\end{array}$$

Thus, $\dfrac{x^4 - 3x^2 + 2}{x^2 - 1} = x^2 - 2, (x \neq \pm 1)$.

48.

$$\begin{array}{r}
3x^2 + 4 \\
x^2 - 1\,\overline{)\,3x^4 + x^2 - 1} \\
\underline{3x^4 - 3x^2} \\
4x^2 - 1 \\
\underline{4x^2 - 4} \\
3
\end{array}$$

Thus, $\dfrac{3x^4 + x^2 - 1}{x^2 - 1} = 3x^2 + 4 + \dfrac{3}{x^2 - 1}$.

49.

$$\begin{array}{r}
5x + 2 \\
x^2 - 3x + 1\,\overline{)\,5x^3 - 13x^2 - x + 2} \\
\underline{5x^3 - 15x^2 + 5x} \\
2x^2 - 6x + 2 \\
\underline{2x^2 - 6x + 2} \\
0
\end{array}$$

Thus, $\dfrac{5x^3 - 13x^2 - x + 2}{x^2 - 3x + 1} = 5x + 2,$

$x \neq \frac{1}{2}(3 \pm \sqrt{5})$.

50.

$$\begin{array}{r}
x^2 - x + 1 \\
x^2 + 2x\,\overline{)\,x^4 + x^3 - x^2 + 2x} \\
\underline{x^4 + 2x^3} \\
-x^3 - x^2 \\
\underline{-x^3 - 2x^2} \\
x^2 + 2x \\
\underline{x^2 + 2x} \\
0
\end{array}$$

Thus,
$\dfrac{x^4 + x^3 - x^2 + 2x}{x^2 + 2x} = x^2 - x + 1, (x \neq 0, -2)$.

51.

$$\begin{array}{r}
3x^2 + 5x + 8 \\
2x^2 + 0x - 1 \,\overline{)\, 6x^4 + 10x^3 + 13x^2 - 5x + 2} \\
\underline{6x^4 + 0x^3 - 3x^2} \phantom{{} - 5x + 2} \\
10x^3 + 16x^2 - 5x \phantom{{} + 2} \\
\underline{10x^3 + 0x^2 - 5x} \phantom{{} + 2} \\
16x^2 - 0 + 2 \\
\underline{16x^2 + 0 - 8} \\
10
\end{array}$$

$$\frac{6x^4 + 10x^3 + 13x^2 - 5x + 2}{2x^2 - 1} = 3x^2 + 5x + 8 + \frac{10}{2x^2 - 1}$$

52.

$$\begin{array}{r}
x^2 - 3x + 2 \\
x^2 + 2 \,\overline{)\, x^4 - 3x^3 + 4x^2 - 6x + 3} \\
\underline{x^4 \phantom{{} - 3x^3} + 2x^2} \phantom{{} - 6x + 3} \\
-3x^3 + 2x^2 - 6x \phantom{{} + 3} \\
\underline{-3x^3 \phantom{{} + 2x^2} - 6x} \phantom{{} + 3} \\
2x^2 \phantom{{} - 6x} + 3 \\
\underline{2x^2 \phantom{{} - 6x} + 4} \\
-1
\end{array}$$

$$\frac{x^4 - 3x^3 + 4x^2 - 6x + 3}{x^2 + 2} = x^2 - 3x + 2 + \frac{-1}{x^2 + 2}$$

53.

$$\begin{array}{r|rrrrr}
-2 & 0.25 & -4 & 0 & 0 & 0 \\
 & & -\frac{1}{2} & 9 & -18 & 36 \\
\hline
 & \frac{1}{4} & -\frac{9}{2} & 9 & -18 & 36
\end{array}$$

Hence,

$$\frac{0.25x^4 - 4x^3}{x + 2} = \frac{1}{4}x^3 - \frac{9}{2}x^2 + 9x - 18 + \frac{36}{x + 2}.$$

54.

$$\begin{array}{r|rrrr}
5 & 0.1 & 0.3 & 0 & -0.5 \\
 & & 0.5 & 4 & 20 \\
\hline
 & 0.1 & 0.8 & 4 & 19.5
\end{array}$$

$$\frac{0.1x^3 + 0.3x^2 - 0.5}{x - 5} = 0.1x^2 + 0.8x + 4 + \frac{19.5}{x - 5}$$

55.

$$\begin{array}{r|rrrrr}
\frac{2}{3} & 6 & -4 & -27 & 18 & 0 \\
 & & 4 & 0 & -18 & 0 \\
\hline
 & 6 & 0 & -27 & 0 & 0
\end{array}$$

Thus,

$$\frac{6x^4 - 4x^3 - 27x^2 + 18x}{x - (2/3)} = 6x^3 - 27x,\ x \neq \frac{2}{3}.$$

56.

$$\begin{array}{r|rrrr}
\frac{1}{2} & 2 & 2 & -1 & 2 \\
 & & 1 & \frac{3}{2} & \frac{1}{4} \\
\hline
 & 2 & 3 & \frac{1}{2} & \frac{9}{4}
\end{array}$$

$$\frac{2x^3 + 2x^2 - x + 2}{x - (1/2)} = 2x^2 + 3x + \frac{1}{2} + \frac{9/4}{x - (1/2)}$$

57.

$$\begin{array}{r|rrrr}
4 & 3 & -10 & 12 & -22 \\
 & & 12 & 8 & 80 \\
\hline
 & 3 & 2 & 20 & 58
\end{array}$$

Thus,

$$\frac{3x^3 - 10x^2 + 12x - 22}{x - 4} = 3x^2 + 2x + 20 + \frac{58}{x - 4}.$$

58.

$$\begin{array}{r|rrrr}
1 & 2 & 6 & -14 & 9 \\
 & & 2 & 8 & -6 \\
\hline
 & 2 & 8 & -6 & 3
\end{array}$$

$$\frac{2x^3 + 6x^2 - 14x + 9}{x - 1} = 2x^2 + 8x - 6 + \frac{3}{x - 1}$$

59. (a)
$$\begin{array}{r|rrrrr} -3 & 1 & 10 & -24 & 20 & 44 \\ & & -3 & -21 & 135 & -465 \\ \hline & 1 & 7 & -45 & 155 & -421 = f(-3) \end{array}$$

(b)
$$\begin{array}{r|rrrrr} -2 & 1 & 10 & -24 & 20 & 44 \\ & & -2 & -16 & 80 & -200 \\ \hline & 1 & 8 & -40 & 100 & -156 = f(-2) \end{array}$$

60. $g(t) = 2t^5 - 5t^4 - 8t + 20$

(a)
$$\begin{array}{r|rrrrrr} -4 & 2 & -5 & 0 & 0 & -8 & 20 \\ & & -8 & 52 & -208 & 832 & -3296 \\ \hline & 2 & -13 & 52 & -208 & 824 & -3276 = g(-4) \end{array}$$

(b)
$$\begin{array}{r|rrrrrr} \sqrt{2} & 2 & -5 & 0 & 0 & -8 & 20 \\ & & 2\sqrt{2} & 4-5\sqrt{2} & 4\sqrt{2}-10 & 8-10\sqrt{2} & -20 \\ \hline & 2 & 2\sqrt{2}-5 & 4-5\sqrt{2} & 4\sqrt{2}-10 & -10\sqrt{2} & 0 \end{array} = g\left(\sqrt{2}\right)$$

61. $f(x) = x^3 + 4x^2 - 25x - 28$

(a)
$$\begin{array}{r|rrrr} 4 & 1 & 4 & -25 & -28 \\ & & 4 & 32 & 28 \\ \hline & 1 & 8 & 7 & 0 \end{array}$$

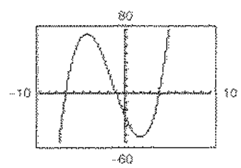
$(x - 4)$ is a factor.

(b) $x^2 + 8x + 7 = (x + 1)(x + 7)$

Remaining factors: $(x + 1)$, $(x + 7)$

(c) $f(x) = (x - 4)(x + 1)(x + 7)$

(d) Zeros: $4, -1, -7$



62. (a) $f(x) = 2x^3 + 11x^2 - 21x - 90$

$$\begin{array}{r|rrrr} -6 & 2 & 11 & -21 & -90 \\ & & -12 & 6 & 90 \\ \hline & 2 & -1 & -15 & 0 \end{array}$$

(b) Remaining factors of $2x^2 - x - 15$ are $(2x + 5)$, $(x - 3)$.

(c) $f(x) = (x + 6)(2x + 5)(x - 3)$

(d) Zeros: $-6, -\frac{5}{2}, 3$

63. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

(a)
$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & & -2 & 12 & -10 & -24 \\ \hline & 1 & -6 & 5 & 12 & 0 \end{array}$$

$(x + 2)$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 5 & 12 \\ & & 3 & -9 & -12 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

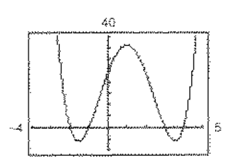
$(x - 3)$ is a factor.

(b) $x^2 - 3x - 4 = (x - 4)(x + 1)$

Remaining factors: $(x - 4)$, $(x + 1)$

(c) $f(x) = (x + 2)(x - 3)(x - 4)(x + 1)$

(d) Zeros: $-2, 3, 4, -1$



64. (a) $f(x) = x^4 - 11x^3 + 41x^2 - 61x + 30$

$$\begin{array}{r|rrrrr} 2 & 1 & -11 & 41 & -61 & 30 \\ & & 2 & -18 & 46 & -30 \\ \hline & 1 & -9 & 23 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -9 & 23 & -15 \\ & & 5 & -20 & 15 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

(b) Remaining factors of $x^2 - 4x + 3$ are $(x - 3)$, $(x - 1)$.

(c) $f(x) = (x - 2)(x - 5)(x - 3)(x - 1)$

(d) Zeros: $1, 2, 3, 5$

65. $f(x) = 4x^3 - 11x^2 + 10x - 3$

Possible rational zeros: $\pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}$

Zeros: $1, 1, \frac{3}{4}$

66. $f(x) = 10x^3 + 21x^2 - x - 6$

Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{5}, \pm\frac{1}{10}, \pm\frac{1}{2}, \pm\frac{3}{10}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{6}{5}, \pm\frac{3}{2}$

Actual rational zeros: $-2, \frac{1}{2}, -\frac{3}{5}$

67. $f(x) = 6x^3 - 5x^2 + 24x - 20$
$= (6x - 5)(x^2 + 4)$

Real zero: $\frac{5}{6}$

68. $f(x) = x^3 - 1.3x^3 - 1.7x + 0.6$
$= \frac{1}{10}(x - 2)(x + 1)(10x - 3)$

Zeros: $-1, 2, \frac{3}{10}$

69. $f(x) = 6x^4 - 25x^3 + 14x^2 + 27x - 18$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

Use a graphing utility to see that $x = -1$ and $x = 3$ are probably zeros.

$$\begin{array}{r|rrrrr} -1 & 6 & -25 & 14 & 27 & -18 \\ & & -6 & 31 & -45 & 18 \\ \hline & 6 & -31 & 45 & -18 & 0 \end{array} \qquad \begin{array}{r|rrrr} 3 & 6 & -31 & 45 & -18 \\ & & 18 & -39 & 18 \\ \hline & 6 & -13 & 6 & 0 \end{array}$$

$$\begin{aligned} 6x^4 - 25x^3 + 14x^2 + 27x - 18 &= (x + 1)(x - 3)(6x^2 - 13x + 6) \\ &= (x + 1)(x - 3)(3x - 2)(2x - 3) \end{aligned}$$

Thus, the zeros of f are $x = -1$, $x = 3$, $x = \frac{2}{3}$, and $x = \frac{3}{2}$.

70. $f(x) = 5x^4 + 126x^2 + 25$
$= (5x^2 + 1)(x^2 + 25)$

No real zeros

71. $g(x) = 5x^3 - 6x + 9$ has two variations in sign $\Longrightarrow$ 0 or 2 positive real zeros.

$g(-x) = -5x^3 + 6x + 9$ has one variation in sign $\Longrightarrow$ 1 negative real zero.

72. $f(x) = 2x^5 - 3x^2 + 2x - 1$ has three variations in sign $\Longrightarrow$ 1 or 3 positive real zeros.

$f(-x) = -2x^5 - 3x^2 - 2x - 1$ has no variations in sign $\Longrightarrow$ 0 negative real zeros.

73.
$$\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & -3 \\ & & 4 & 1 & 5 \\ \hline & 4 & 1 & 5 & 2 \end{array}$$

All entries positive; $x = 1$ is upper bound.

$$\begin{array}{r|rrrr} -\frac{1}{4} & 4 & -3 & 4 & -3 \\ & & -1 & 1 & -\frac{5}{4} \\ \hline & 4 & -4 & 5 & -\frac{17}{4} \end{array}$$

Alternating signs; $x = -\frac{1}{4}$ is lower bound.

74.
$$\begin{array}{r|rrrr} 8 & 2 & -5 & -14 & 8 \\ & & 16 & 88 & 592 \\ \hline & 2 & 11 & 74 & 600 \end{array}$$

All positive $\Longrightarrow$ $x = 8$ is upper bound.

$$\begin{array}{r|rrrr} -4 & 2 & -5 & -14 & 8 \\ & & -8 & 52 & -152 \\ \hline & 2 & -13 & 38 & -144 \end{array}$$

Alternating signs $\Longrightarrow$ $x = -4$ is lower bound.

75. $6 + \sqrt{-25} = 6 + 5i$

76. $-\sqrt{-12} + 3 = -2\sqrt{3}i + 3 = 3 - 2\sqrt{3}i$

77. $-2i^2 + 7i = 2 + 7i$

78. $-i^2 - 4i = 1 - 4i$

79. $(7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i)$
$= 3 + 7i$

80. $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = -\sqrt{2}\,i$

81. $5i(13 - 8i) = 65i - 40i^2 = 40 + 65i$

82. $(1 + 6i)(5 - 2i) = 5 - 2i + 30i + 12 = 17 + 28i$

83. $\left(\sqrt{-16} + 3\right)\left(\sqrt{-25} - 2\right) = (4i + 3)(5i - 2)$
$= -20 - 8i + 15i - 6$
$= -26 + 7i$

84. $\left(5 - \sqrt{-4}\right)\left(5 + \sqrt{-4}\right) = (5 - 2i)(5 + 2i)$
$= 25 + 4$
$= 29$

85. $\sqrt{-9} + 3 + \sqrt{-36} = 3i + 3 + 6i$
$= 3 + 9i$

86. $7 - \sqrt{-81} + \sqrt{-49} = 7 - 9i + 7i$
$= 7 - 2i$

87. $(10 - 8i)(2 - 3i) = 20 - 30i - 16i + 24i^2$
$= -4 - 46i$

88. $i(6 + i)(3 - 2i) = i(18 + 3i - 12i + 2)$
$= i(20 - 9i) = 9 + 20i$

89. $(3 + 7i)^2 + (3 - 7i)^2 = (9 + 42i - 49) + (9 - 42i - 49)$
$= -80$

90. $(4 - i)^2 - (4 + i)^2 = (16 - 8i - 1) - (16 + 8i - 1)$
$= -16i$

91. $\frac{6 + i}{i} = \frac{6 + i}{i} \cdot \frac{-i}{-i} = \frac{-6i - i^2}{-i^2}$
$= \frac{-6i + 1}{1} = 1 - 6i$

92. $\frac{4}{-3i} = \frac{-4}{3i} \cdot \frac{-i}{-i} = \frac{4i}{3} = \frac{4}{3}i$

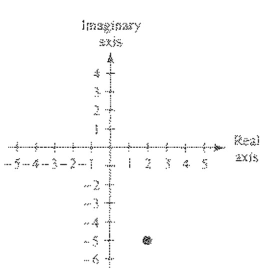
93. $\frac{3 + 2i}{5 + i} \cdot \frac{5 - i}{5 - i} = \frac{15 + 10i - 3i + 2}{25 + 1}$
$= \frac{17}{26} + \frac{7}{26}i$

94. $\frac{1 - 7i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} = \frac{2 - 21 - 17i}{4 + 9}$
$= \frac{-19}{13} + \frac{-17}{13}i$

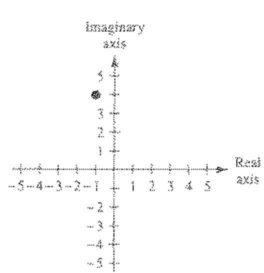
95. $-3 - 2i$

96. $2 - i$

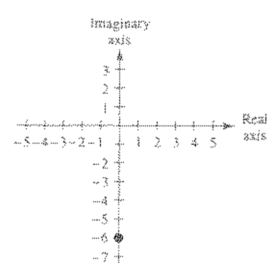
97. $2 - 5i$



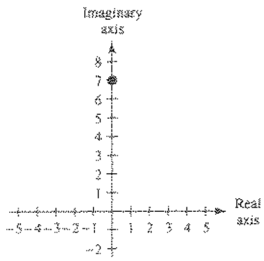
98. $-1 + 4i$



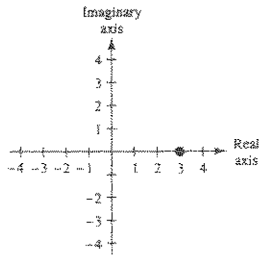
99. $-6i$



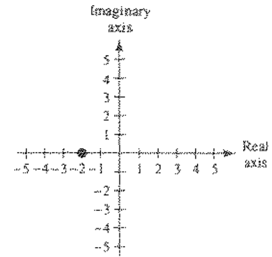
100. $7i$



101. 3



102. -2



103. $f(x) = 3x(x-2)^2$

Zeros: $0, 2, 2$

104. $f(x) = (x-4)(x+9)^2$

Zeros: $4, -9, -9$

105. $f(x) = 2x^4 - 5x^3 + 10x - 12$

$$\begin{array}{r|rrrrr} 2 & 2 & -5 & 0 & 10 & -12 \\ & & 4 & -2 & -4 & 12 \\ \hline & 2 & -1 & -2 & 6 & 0 \end{array}$$

$x = 2$ is a zero.

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & -1 & -2 & 6 \\ & & -3 & 6 & -6 \\ \hline & 2 & -4 & 4 & 0 \end{array}$$

$x = -\dfrac{3}{2}$ is a zero.

$$f(x) = (x-2)\left(x+\frac{3}{2}\right)(2x^2 - 4x + 4)$$

$$= (x-2)(2x+3)(x^2 - 2x + 2)$$

By the Quadratic Formula, applied to $x^2 - 2x + 2$,

$$x = \frac{2 \pm \sqrt{4 - 4(2)}}{2} = 1 \pm i.$$

Zeros: $2, -\dfrac{3}{2}, 1 \pm i$

$$f(x) = (x-2)(2x+3)(x-1+i)(x-1-i)$$

106. $g(x) = 3x^4 - 4x^3 + 7x^2 + 10x - 4$

$$= (x+1)(3x-1)(x^2 - 2x + 4)$$

Quadratic: $x = \dfrac{2 \pm \sqrt{4-16}}{2} = 1 \pm \sqrt{3}i$

Zeros: $-1, \dfrac{1}{3}, 1 \pm \sqrt{3}i$

$$g(x) = (x+1)(3x-1)\left(x - 1 + \sqrt{3}i\right)\left(x - 1 - \sqrt{3}i\right)$$

107. $h(x) = x^3 - 7x^2 + 18x - 24$

$$\begin{array}{r|rrrr} 4 & 1 & -7 & 18 & -24 \\ & & 4 & -12 & 24 \\ \hline & 1 & -3 & 6 & 0 \end{array}$$

$x = 4$ is a zero. Applying the Quadratic Formula on $x^2 - 3x + 6$,

$$x = \frac{3 \pm \sqrt{9 - 4(6)}}{2} = \frac{3}{2} \pm \frac{\sqrt{15}}{2}i.$$

Zeros: $4, \dfrac{3}{2} + \dfrac{\sqrt{15}}{2}i, \dfrac{3}{2} - \dfrac{\sqrt{15}}{2}i$

$$h(x) = (x-4)\left(x - \frac{3+\sqrt{15}i}{2}\right)\left(x - \frac{3-\sqrt{15}i}{2}\right)$$

108. $f(x) = 2x^3 - 5x^2 - 9x + 40$

$$= (2x+5)(x^2 - 5x + 8)$$

Quadratic: $x = \dfrac{5 \pm \sqrt{25-32}}{2} = \dfrac{5 \pm \sqrt{7}i}{2}$

Zeros: $-\dfrac{5}{2}, \dfrac{5}{2} \pm \dfrac{\sqrt{7}}{2}i$

$$f(x) = (2x+5)\left(x - \frac{5}{2} + \frac{\sqrt{7}}{2}i\right)\left(x - \frac{5}{2} - \frac{\sqrt{7}}{2}i\right)$$

109. $f(x) = x^5 + x^4 + 5x^3 + 5x^2$

$= x^2(x^3 + x^2 + 5x + 5)$

$= x^2[x^2(x + 1) + 5(x + 1)]$

$= x^2(x + 1)(x^2 + 5)$

$= x^2(x + 1)(x + \sqrt{5}i)(x - \sqrt{5}i)$

Zeros: $0, 0, -1, \pm\sqrt{5}i$

110. $f(x) = x^5 - 5x^3 + 4x$

$= x(x^4 - 5x^2 + 4)$

$= x(x^2 - 4)(x^2 - 1)$

$f(x) = x(x - 2)(x + 2)(x - 1)(x + 1)$

Zeros: $0, \pm 1, \pm 2$

111. $f(x) = x^3 - 4x^2 + 6x - 4$

(a) $x^3 - 4x^2 + 6x - 4 = (x - 2)(x^2 - 2x + 2)$

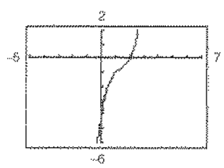
By the Quadratic Formula, for $x^2 - 2x + 2$,

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)}}{2} = 1 \pm i.$$

Zeros: $2, 1 + i, 1 - i$

(b) $f(x) = (x - 2)(x - 1 - i)(x - 1 + i)$

(c) x-intercept: $(2, 0)$



112. (a) $f(x) = x^3 - 5x^2 - 7x + 51$

$= (x + 3)(x^2 - 8x + 17)$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(17)}}{2} = \frac{8 \pm \sqrt{-4}}{2}$$

$= 4 \pm i$

Zeros: $-3, 4 + i, 4 - i$

(b) $f(x) = (x + 3)(x - 4 - i)(x - 4 + i)$

(c) x-intercept: $(-3, 0)$

113. (a) $f(x) = -3x^3 - 19x^2 - 4x + 12$

$$\begin{array}{r|rrrr} -1 & -3 & -19 & -4 & 12 \\ & & 3 & 16 & -12 \\ \hline & -3 & -16 & 12 & 0 \end{array}$$

(b) $f(x) = -(x + 1)(3x^2 + 16x - 12)$

$= -(x + 1)(3x - 2)(x + 6)$

(c) x-intercepts: $(-1, 0), (-6, 0), \left(\frac{2}{3}, 0\right)$

114. (a) $f(x) = 2x^3 - 9x^2 + 22x - 30$

$= (2x - 5)(x^2 - 2x + 6)$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(6)}}{2} = 1 \pm \sqrt{5}i$$

Zeros: $\frac{5}{2}, 1 + \sqrt{5}i, 1 - \sqrt{5}i$

(b) $f(x) = (2x - 5)(x - 1 - \sqrt{5}i)(x - 1 + \sqrt{5}i)$

(c) x-intercept: $\left(\frac{5}{2}, 0\right)$

115. $f(x) = x^4 + 34x^2 + 225$

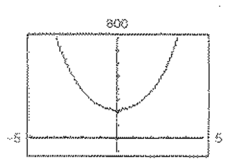
(a) $x^4 + 34x^2 + 225 = (x^2 + 9)(x^2 + 25)$

Zeros: $\pm 3i, \pm 5i$

(b) $(x + 3i)(x - 3i)(x + 5i)(x - 5i)$

(c) No x-intercepts

(d)



116. (a), (b)

$f(x) = x^4 + 10x^3 + 26x^2 + 10x + 25$

$= (x^2 + 1)(x^2 + 10x + 25)$

$= (x^2 + 1)(x + 5)^2 = (x + i)(x - i)(x + 5)^2$

Zeros: $\pm i, -5, -5$

(c) x-intercept: $(-5, 0)$

117. Since $5i$ is a zero, so is $-5i$.

$$\begin{aligned} f(x) &= (x-4)(x+2)(x-5i)(x+5i) \\ &= (x^2-2x-8)(x^2+25) \\ &= x^4-2x^3+17x^2-50x-200 \end{aligned}$$

118. Since $2i$ is a zero, so is $-2i$.

$$\begin{aligned} f(x) &= (x-2)(x+2)(x-2i)(x+2i) \\ &= (x^2-4)(x^2+4) \\ &= x^4-16 \end{aligned}$$

119. $$\begin{aligned} f(x) &= (x-1)(x+4)(x+3-5i)(x+3+5i) \\ &= (x^2+3x-4)((x+3)^2+25) \\ &= (x^2+3x-4)(x^2+6x+34) \\ &= x^4+9x^3+48x^2+78x-136 \end{aligned}$$

120. $$\begin{aligned} f(x) &= (x+4)(x+4)\left(x-1-\sqrt{3}i\right)\left(x-1+\sqrt{3}i\right) \\ &= (x^2+8x+16)((x-1)^2+3) \\ &= (x^2+8x+16)(x^2-2x+4) \\ &= x^4+6x^3+4x^2+64 \end{aligned}$$

121. $f(x) = x^4-2x^3+8x^2-18x-9$

(a) $f(x) = (x^2+9)(x^2-2x-1)$

For the quadratic x^2-2x-1, $x = \dfrac{2 \pm \sqrt{(-2)^2-4(-1)}}{2} = 1 \pm \sqrt{2}$.

(b) $f(x) = (x^2+9)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$

(c) $f(x) = (x+3i)(x-3i)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$

122. $f(x) = x^4-4x^3+3x^2+8x-16$

(a) $f(x) = (x^2-x-4)(x^2-3x+4)$

(b) $x = \dfrac{1 \pm \sqrt{(-1)^2-4(-4)}}{2} = \dfrac{1}{2} \pm \dfrac{\sqrt{17}}{2}$

$$f(x) = \left(x-\frac{1}{2}-\frac{\sqrt{17}}{2}\right)\left(x-\frac{1}{2}+\frac{\sqrt{17}}{2}\right)(x^2-3x+4)$$

(c) $x = \dfrac{3 \pm \sqrt{(-3)^2-4(4)}}{2} = \dfrac{3}{2} \pm \dfrac{\sqrt{7}}{2}i$

$$f(x) = \left(x-\frac{1}{2}-\frac{\sqrt{17}}{2}\right)\left(x-\frac{1}{2}+\frac{\sqrt{17}}{2}\right)\left(x-\frac{3}{2}+\frac{\sqrt{7}}{2}i\right)\left(x-\frac{3}{2}-\frac{\sqrt{7}}{2}i\right)$$

123. Zeros: $-2i, 2i$

$(x+2i)(x-2i) = x^2+4$ is a factor.

$f(x) = (x^2+4)(x+3)$

Zeros: $\pm 2i, -3$

124. Zeros: $2+\sqrt{5}i, 2-\sqrt{5}i$

$\left(x-2-\sqrt{5}i\right)\left(x-2+\sqrt{5}i\right) = (x-2)^2+5 = x^2-4x+9$ is a factor.

$f(x) = (x^2-4x+9)(2x+1)$

Zeros: $x = -\frac{1}{2}, 2 \pm \sqrt{5}i$

125. (a) Domain: all $x \neq -3$

(b) Horizontal asymptote: $y = -1$

Vertical asymptote: $x = -3$

126. (a) Domain: all $x \neq 8$

(b) Horizontal asymptote: $y = 4$

Vertical asymptote: $x = 8$

127. $f(x) = \dfrac{2}{x^2 - 3x - 18} = \dfrac{2}{(x-6)(x+3)}$

(a) Domain: all $x \neq 6, -3$

(b) Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = 6,\ x = -3$

128. The denominator $x^2 + x + 3$ has no zeros.

Domain: all x

Horizontal asymptote: $y = 2$

Vertical asymptotes: none

129. $f(x) = \dfrac{7 + x}{7 - x}$

(a) Domain: all $x \neq 7$

(b) Horizontal asymptote: $y = -1$

Vertical asymptote: $x = 7$

130. $f(x) = \dfrac{6x}{x^2 - 1} = \dfrac{6x}{(x+1)(x-1)}$

(a) Domain: all $x \neq \pm 1$

(b) Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = \pm 1$

131. $f(x) = \dfrac{4x^2}{2x^2 - 3}$

(a) Domain: all $x \neq \pm\sqrt{\dfrac{3}{2}} = \pm\dfrac{\sqrt{6}}{2}$

(b) Horizontal asymptote: $y = 2$

Vertical asymptotes: $x = \pm\sqrt{\dfrac{3}{2}} = \pm\dfrac{\sqrt{6}}{2}$

132. $f(x) = \dfrac{3x^2 - 11x - 4}{x^2 + 2}$

(a) Domain: all x

(b) Horizontal asymptote: $y = 3$

No vertical asymptote

133. $f(x) = \dfrac{2x - 10}{x^2 - 2x - 15} = \dfrac{2(x-5)}{(x-5)(x+3)} = \dfrac{2}{x+3},$

$x \neq 5$

(a) Domain: all $x \neq 5, -3$

(b) Vertical asymptote: $x = -3$

(There is a hole at $x = 5$.)

Horizontal asymptote: $y = 0$

134. $f(x) = \dfrac{x^3 - 4x^2}{x^2 + 3x + 2} = \dfrac{x^2(x-4)}{(x+2)(x+1)}$

(a) Domain: all $x \neq -1, -2$

(b) Vertical asymptote: $x = -2, x = -1$

No horizontal asymptotes

135. $f(x) = \dfrac{x - 2}{|x| + 2}$

(a) Domain: all real numbers

(b) No vertical asymptotes

Horizontal asymptotes: $y = 1, y = -1$

136. $f(x) = \dfrac{2x}{|2x - 1|}$

(a) Domain: all $x \neq \dfrac{1}{2}$

(b) Vertical asymptote: $x = \dfrac{1}{2}$

Horizontal asymptotes: $y = 1$ (to the right)
$y = -1$ (to the left)

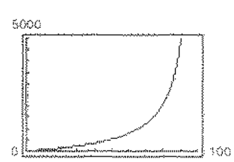
137. $C = \dfrac{528p}{100 - p},\quad 0 \leq p < 100$

(a) When $p = 25$, $C = \dfrac{528(25)}{100 - 25} = 176$ million.

When $p = 50$, $C = \dfrac{528(50)}{100 - 50} = 528$ million.

When $p = 75$, $C = \dfrac{528(75)}{100 - 75} = 1584$ million.

(b)



(c) No. As $p \to 100$, C tends to infinity.

138. $y = \dfrac{1.568x - 0.001}{6.360x + 1}, x > 0$

The moth will be satiated at the horizontal asymptote, $y = \dfrac{1.568}{6.360} \approx 0.247$ mg.

139. $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 1}$

$= \dfrac{(x - 4)(x - 1)}{(x - 1)(x + 1)}$

$= \dfrac{x - 4}{x + 1}, \quad x \neq 1$

Vertical asymptotes: $x = -1$

Horizontal asymptote: $y = 1$

No slant asymptotes

Hole at $x = 1$: $\left(1, -\dfrac{3}{2}\right)$

140. $f(x) = \dfrac{x^2 - 3x - 8}{x^2 - 4}$

$= \dfrac{x^2 - 3x - 8}{(x - 2)(x + 2)}$

Vertical asymptotes: $x = \pm 2$

Horizontal asymptote: $y = 1$

No slant asymptotes

Holes: none

141. $f(x) = \dfrac{2x^2 - 7x + 3}{2x^2 - 3x - 9}$

$= \dfrac{(x - 3)(2x - 1)}{(x - 3)(2x + 3)}$

$= \dfrac{2x - 1}{2x + 3}, x \neq 3$

Vertical asymptote: $x = -\dfrac{3}{2}$

Horizontal asymptote: $y = 1$

No slant asymptotes

Hole at $x = 3$: $\left(3, \dfrac{5}{9}\right)$

142. $f(x) = \dfrac{3x^2 + 13x - 10}{2x^2 + 11x + 5}$

$= \dfrac{(x + 5)(3x - 2)}{(x + 5)(2x + 1)}$

$= \dfrac{3x - 2}{2x + 1}, \quad x \neq -5$

Vertical asymptote: $x = -\dfrac{1}{2}$

Horizontal asymptote: $y = \dfrac{3}{2}$

No slant asymptotes

Hole at $x = -5$: $\left(-5, \dfrac{17}{9}\right)$

143. $f(x) = \dfrac{3x^3 - x^2 - 12x + 4}{x^2 + 3x + 2}$

$= \dfrac{(x - 2)(x + 2)(3x - 1)}{(x + 1)(x + 2)}$

$= \dfrac{(x - 2)(3x - 1)}{x + 1}, \quad x \neq -2$

$= 3x - 10 + \dfrac{12}{x + 1}, \quad x \neq -2$

Vertical asymptote: $x = -1$

No horizontal asymptotes

Slant asymptote: $y = 3x - 10$

Hole at $x = -2$: $(-2, -28)$

144. $f(x) = \dfrac{2x^3 + 3x^2 - 2x - 3}{x^2 - 3x + 2}$

$= \dfrac{(x - 1)(x + 1)(2x + 3)}{(x - 1)(x - 2)}$

$= \dfrac{(x + 1)(2x + 3)}{x - 2}, \quad x \neq 1$

$= 2x + 9 + \dfrac{21}{x - 2}, \quad x \neq 1$

Vertical asymptote: $x = 2$

No horizontal asymptotes

Slant asymptote: $y = 2x + 9$

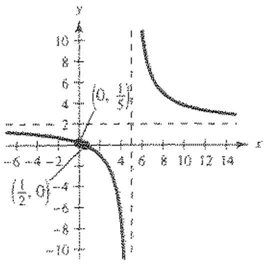
Hole at $x = 1$: $(1, -10)$

145. $f(x) = \dfrac{2x - 1}{x - 5}$

Intercepts: $\left(0, \frac{1}{5}\right), \left(\frac{1}{2}, 0\right)$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = 2$



146. $f(x) = \dfrac{x - 3}{x - 2}$

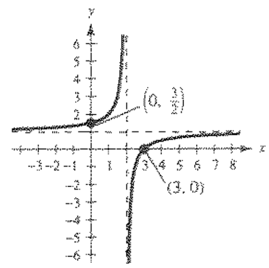
x-intercept: $(3, 0)$

y-intercept: $\left(0, \frac{3}{2}\right)$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

x	-1	0	1	3	4	5
y	$\frac{4}{3}$	$\frac{3}{2}$	2	0	$\frac{1}{2}$	$\frac{2}{3}$



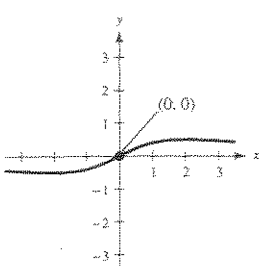
147. $f(x) = \dfrac{2x}{x^2 + 4}$

Intercept: $(0, 0)$

Origin symmetry

Horizontal asymptote: $y = 0$

x	-2	-1	0	1	2
y	$-\frac{1}{2}$	$-\frac{2}{5}$	0	$\frac{2}{5}$	$\frac{1}{2}$



148. $f(x) = \dfrac{2x^2}{x^2 - 4}$

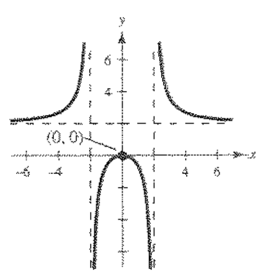
Intercept: $(0, 0)$

y-axis symmetry

Vertical asymptotes: $x = 2$, $x = -2$

Horizontal asymptote: $y = 2$

x	± 5	± 4	± 3	± 1	0
y	$\frac{50}{21}$	$\frac{8}{3}$	$\frac{18}{5}$	$-\frac{2}{3}$	0



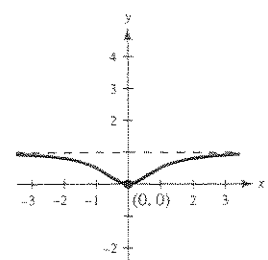
149. $f(x) = \dfrac{x^2}{x^2 + 1}$

Intercept: $(0, 0)$

y-axis symmetry

Horizontal asymptote: $y = 1$

x	± 3	± 2	± 1	0
y	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{1}{2}$	0



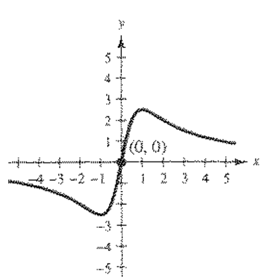
150. $f(x) = \dfrac{5x}{x^2 + 1}$

Intercept: $(0, 0)$

Symmetry: origin

Horizontal asymptote: $y = 0$

No vertical asymptotes

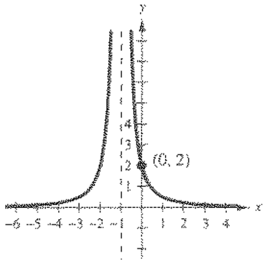


151. $f(x) = \dfrac{2}{(x+1)^2}$

Intercept: $(0, 2)$

Horizontal asymptote: $y = 0$

Vertical asymptote: $x = -1$



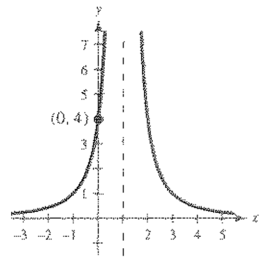
152. $h(x) = \dfrac{4}{(x-1)^2}$

y-intercept: $(0, 4)$

Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 0$

x	-2	-1	0	2	3	4
y	$\frac{4}{9}$	1	4	4	1	$\frac{4}{9}$



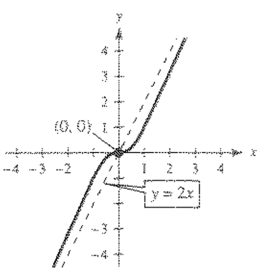
153. $f(x) = \dfrac{2x^3}{x^2+1} = 2x - \dfrac{2x}{x^2+1}$

Intercept: $(0, 0)$

Origin symmetry

Slant asymptote: $y = 2x$

x	-2	-1	0	1	2
y	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$

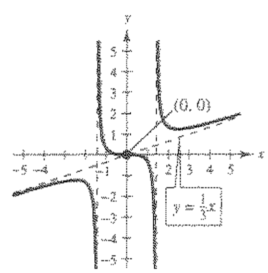


154. $f(x) = \dfrac{x^3}{3x^2-6} = \dfrac{1}{3}x + \dfrac{2x}{3x^2-6} = \dfrac{1}{3}\left[x + \dfrac{2x}{x^2-2}\right]$

Intercepts: $(0, 0)$

Vertical asymptotes: $x = \pm\sqrt{2}$

Slant asymptote: $y = \dfrac{1}{3}x$

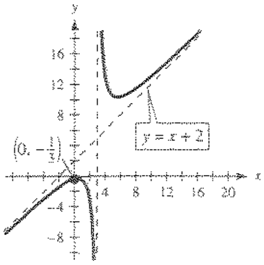


155. $f(x) = \dfrac{x^2-x+1}{x-3} = x + 2 + \dfrac{7}{x-3}$

Intercept: $\left(0, -\dfrac{1}{3}\right)$

Vertical asymptote: $x = 3$

Slant asymptote: $y = x + 2$

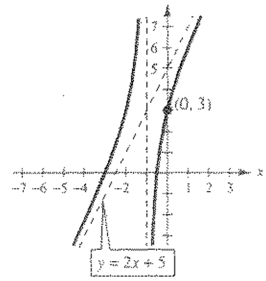


156. $f(x) = \dfrac{2x^2+7x+3}{x+1} = 2x + 5 - \dfrac{2}{x+1}$

Intercepts: $(0, 3), (-3, 0), \left(-\dfrac{1}{2}, 0\right)$

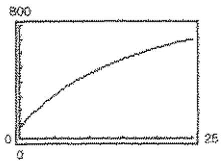
Vertical asymptote: $x = -1$

Slant asymptote: $y = 2x + 5$



157. $N = \dfrac{20(4 + 3t)}{1 + 0.05t}, \quad t \geq 0$

(a)



(b) $N(5) = 304{,}000$ fish

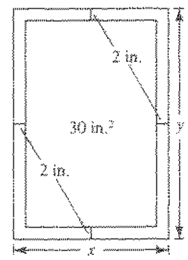
$N(10) \approx 453{,}333$ fish

$N(25) \approx 702{,}222$ fish

(c) The limit is

$\dfrac{60}{0.05} = 1{,}200{,}000$ fish, the horizontal asymptote.

158. (a)



(b) $(x - 4)(y - 4) = 30 \Rightarrow y = 4 + \dfrac{30}{x - 4}$

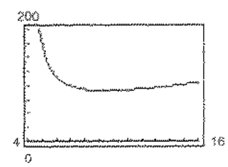
$$\text{Area} = A = xy = x\left[4 + \frac{30}{x - 4}\right]$$

$$= x\left[\frac{4x - 16 + 30}{x - 4}\right]$$

$$= \frac{2x(2x + 7)}{x - 4}$$

(c) Domain: $x > 4$

(d)



9.48 by 9.48

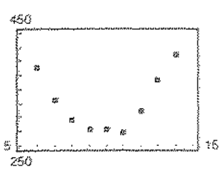
159. Quadratic model

160. Neither

161. Linear model

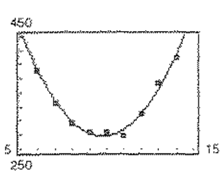
162. Quadratic

163. (a)



(b) $y = 8.03t^2 - 157.1t + 1041$; 0.98348

(c)

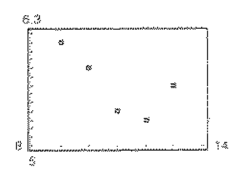


Yes, the model is a good fit.

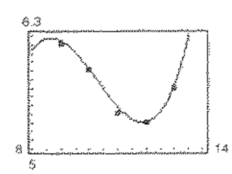
(d) From the model, $y \geq 500$ when $t \approx 15.1$, or 2005.

(e) Answers will vary.

164. (a)



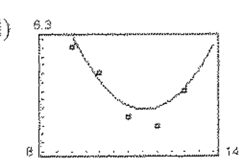
(b)



Yes, the model is an excellent fit.

(c) $y = 0.129t^2 - 2.99t + 22.8$

(d)



The quadratic model is not a good fit.

(e) The cubic model is better.

(f) Answers will vary.

165. False. The degree of the numerator is two more than the degree of the denominator.

166. False. A fourth degree polynomial with real coefficients can have at most four zeros. Since $-8i$ and $4i$ are zeros, so are $8i$ and $-4i$.

167. False. $(1 + i) + (1 - i) = 2$, a real number

168. It means that the divisor is a factor of the dividend.

169. Not every rational function has a vertical asymptote. For example,

$$y = \frac{x}{x^2 + 1}.$$

170. $\sqrt{-6}\sqrt{-6} \neq \sqrt{(-6)(-6)}$

In fact, $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\,\sqrt{6}i = -6$.

171. The error is $\sqrt{-4} \neq 4i$. In fact,

$-i\left(\sqrt{-4} - 1\right) = -i(2i - 1) = 2 + i$.

172. (a) $i^{40} = (i^4)^{10} = 1^{10} = 1$

(b) $i^{25} = i(i^{24}) = i(1) = i$

(c) $i^{50} = i^2(i^{48}) = (-1)(1) = -1$

(d) $i^{67} = i^3(i^{64}) = -i(1) = -i$

Chapter 2 Practice Test

1. Sketch the graph of $f(x) = x^2 - 6x + 5$ by hand and identify the vertex and the intercepts.

2. Find the number of units x that produce a minimum cost C if $C = 0.01x^2 - 90x + 15{,}000$.

3. Find the quadratic function that has a maximum at $(1, 7)$ and passes through the point $(2, 5)$.

4. Find two quadratic functions that have x-intercepts $(2, 0)$ and $\left(\frac{4}{3}, 0\right)$.

5. Use the leading Coefficient Test to determine the right-hand and left-hand behavior of the graph of the polynomial function $f(x) = -3x^5 + 2x^3 - 17$.

6. Find all the real zeros of $f(x) = x^5 - 5x^3 + 4x$. Verify your answer with a graphing utility.

7. Find a polynomial function with 0, 3, and -2 as zeros.

8. Sketch $f(x) = x^3 - 12x$ by hand.

9. Divide $3x^4 - 7x^2 + 2x - 10$ by $x - 3$ using long division.

10. Divide $x^3 - 11$ by $x^2 + 2x - 1$.

11. Use synthetic division to divide $3x^5 + 13x^4 + 12x - 1$ by $x + 5$.

12. Use synthetic division to find $f(-6)$ when $f(x) = 7x^3 + 40x^2 - 12x + 15$.

13. Find the real zeros of $f(x) = x^3 - 19x - 30$.

14. Find the real zeros of $f(x) = x^4 + x^3 - 8x^2 - 9x - 9$.

15. List all possible rational zeros of the function $f(x) = 6x^3 - 5x^2 + 4x - 15$.

16. Find the rational zeros of the polynomial $f(x) = x^3 - \frac{20}{3}x^2 + 9x - \frac{10}{3}$.

17. Write $f(x) = x^4 + x^3 + 3x^2 + 5x - 10$ as a product of linear factors.

18. Write $\dfrac{2}{1 + i}$ in standard form.

19. Write $\dfrac{3 + i}{2} - \dfrac{i + 1}{4}$ in standard form.

20. Find a polynomial with real coefficients that has 2, $3 + i$, and $3 - 2i$ as zeros.

21. Use synthetic division to show that $3i$ is a zero of $f(x) = x^3 + 4x^2 + 9x + 36$.

22. Find a mathematical model for the statement, "z varies directly as the square of x and inversely as the square root of y".

23. Sketch the graph of $f(x) = \dfrac{x - 1}{2x}$ and label all intercepts and asymptotes.

24. Sketch the graph of $f(x) = \dfrac{3x^2 - 4}{x}$ and label all intercepts and asymptotes.

25. Find all the asymptotes of $f(x) = \dfrac{8x^2 - 9}{x^2 + 1}$.

26. Find all the asymptotes of $f(x) = \dfrac{4x^2 - 2x + 7}{x - 1}$.

27. Sketch the graph of $f(x) = \dfrac{x - 5}{(x - 5)^2}$.

CHAPTER 3
Exponential and Logarithmic Functions